

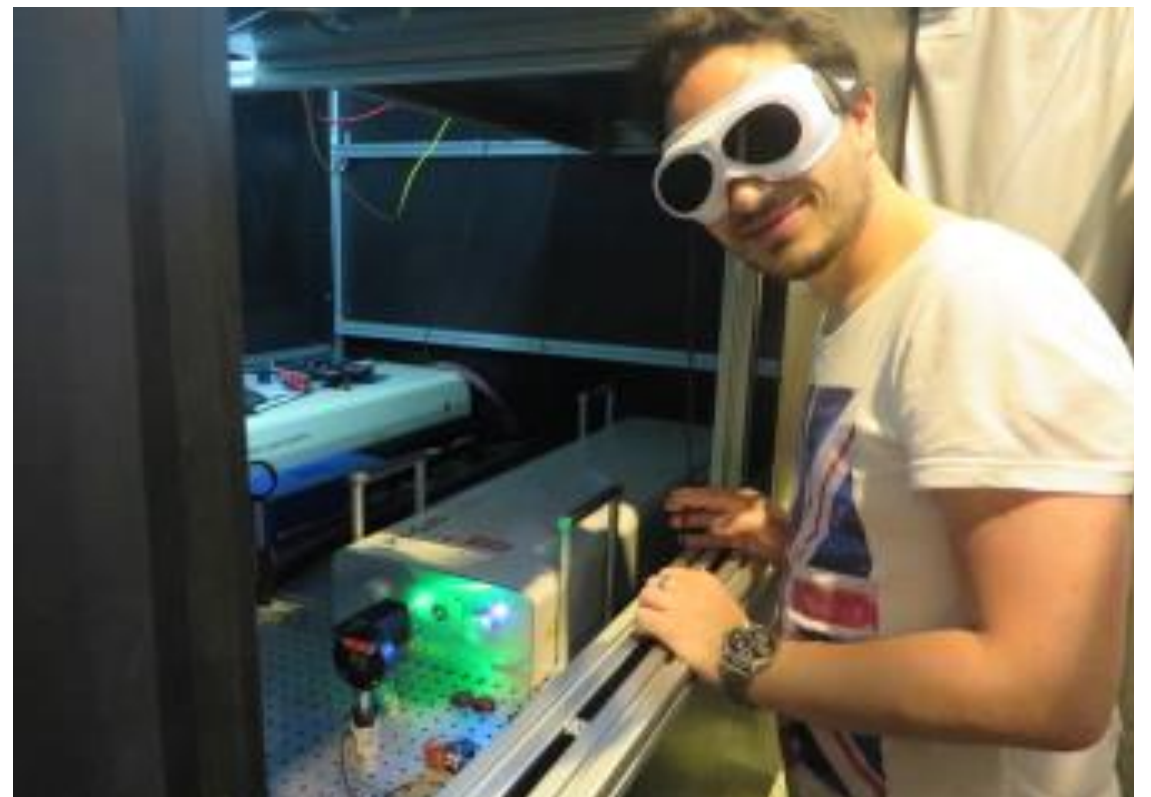
Laser-assisted modern nuclear physics

Lecture 1:

From the atom to the nucleus
& the production of radioactive ion beams

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Who is Dr Thomas?

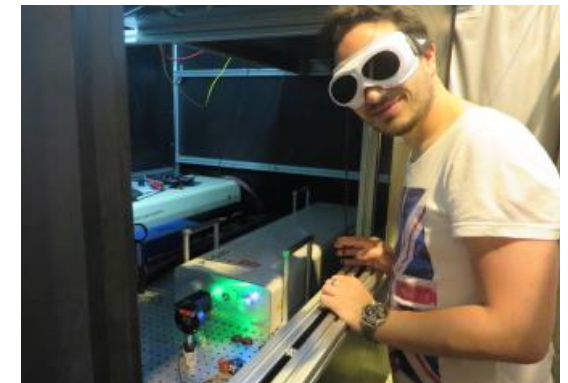
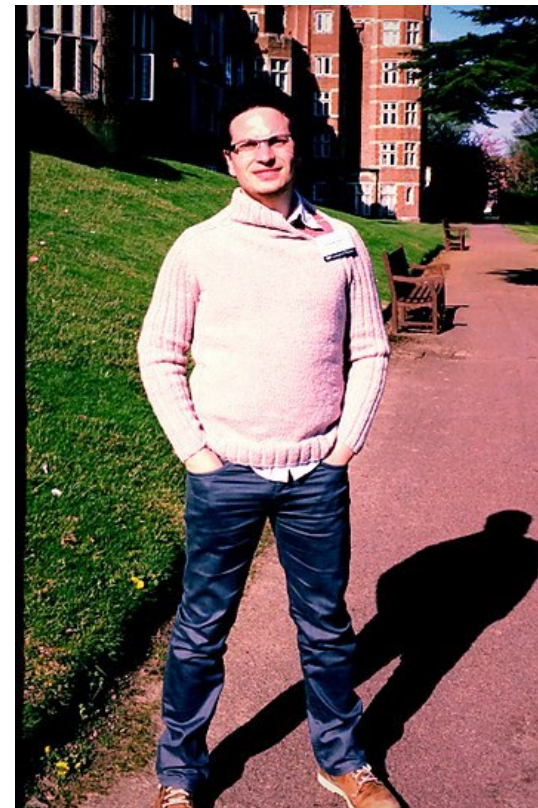
French - Greek - American

B.Sc.'03, M.Sc.'05, McGill
University, Montréal, Qc

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Who is Dr Thomas?

Keywords

Laser

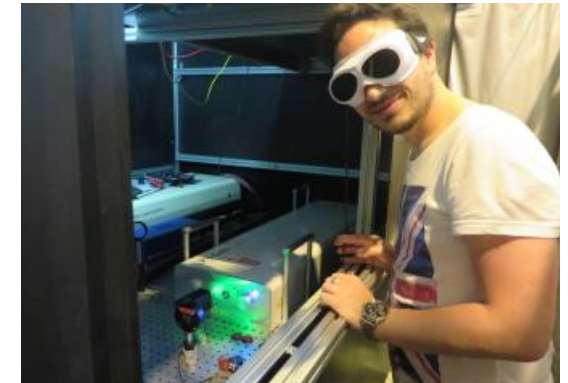
Polonium

Out of the box

Knitting

Singing

Running



Laser-assisted modern nuclear physics

- Lecture 1:
 - ▶ Fundamentals of the atom-nucleus interaction
 - ▶ Lasers for the production of radioactive ion beams
- Lecture 2:
 - ▶ High-resolution collinear laser spectroscopy
 - ▶ Atom trapping
 - ▶ Anti-atomic studies



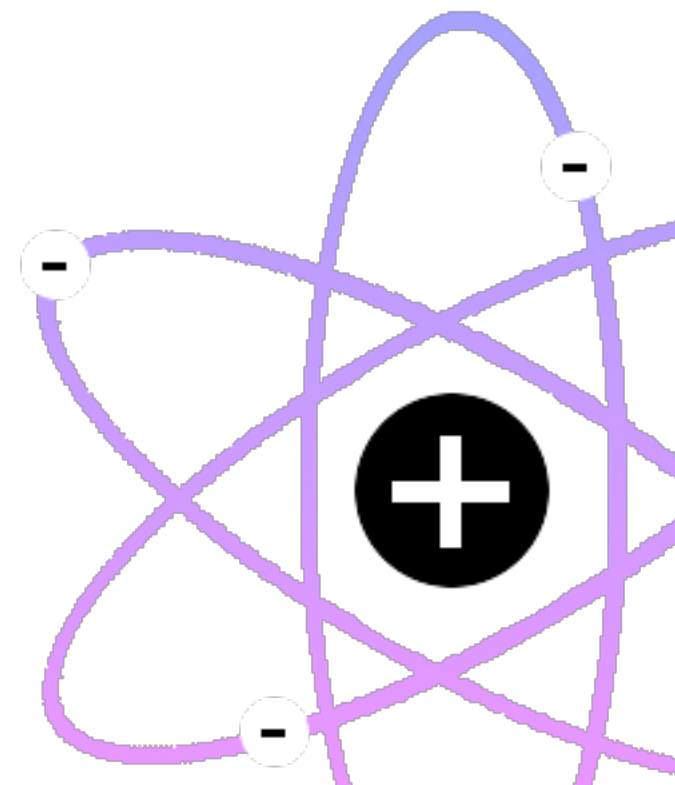
The atom

A quantum system under EM forces

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = E\psi.$$

$$V(r) = \frac{Ze^2}{4\pi\epsilon_0 r}$$

Let's enjoy some math now!



Solving the (hydrogen) atom

Double separation of variables in spherical coordinates

$$\psi(r, \theta, \phi) = R(r)Y(\theta, \phi)$$

$$\frac{1}{R(r)} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) R(r) - \frac{2mr^2}{\hbar^2} [V(r) - E] = l(l+1),$$

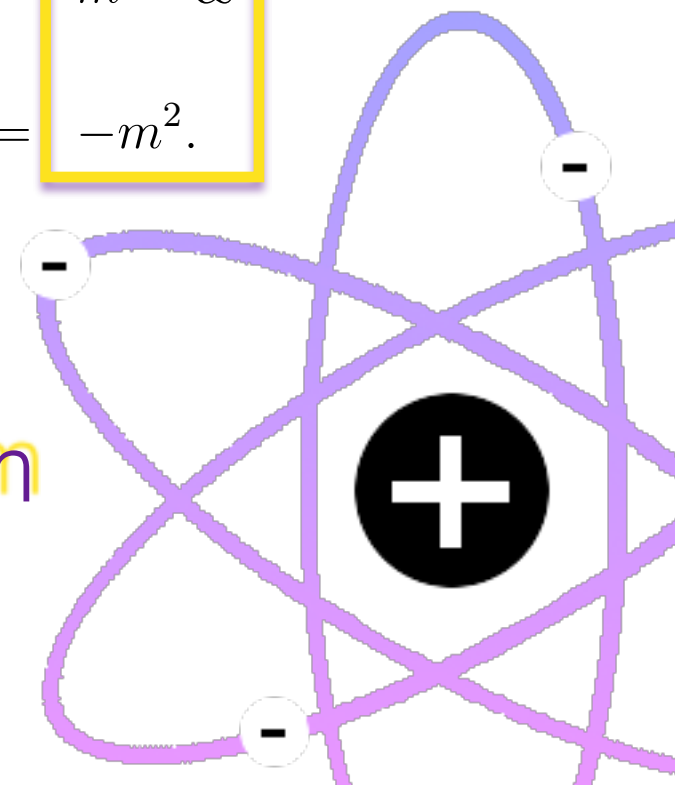
$$\frac{1}{Y(\theta, \phi) \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) Y(\theta, \phi) + \frac{1}{Y(\theta, \phi) \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} Y(\theta, \phi) = -l(l+1).$$

$$Y(\theta, \phi) = f(\theta)g(\phi)$$

$$\frac{\sin \theta}{f(\theta)} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \right) f(\theta) + l(l+1) \sin^2 \theta = m^2 \quad \&$$

$$\frac{1}{g(\phi)} \frac{d^2}{d\phi^2} g(\phi) = -m^2.$$

Separation constants make angular momentum quantum numbers appear naturally



Solving the (hydrogen) atom

Solving backwards reveals some conditions on l & m

$$g_m(\phi) = e^{im\phi}$$

$$f_{l,m}(\theta) = (-1)^m \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_{l,m}(\cos\theta),$$

where $l \in \mathbb{N}$ & $|m| \leq l$.

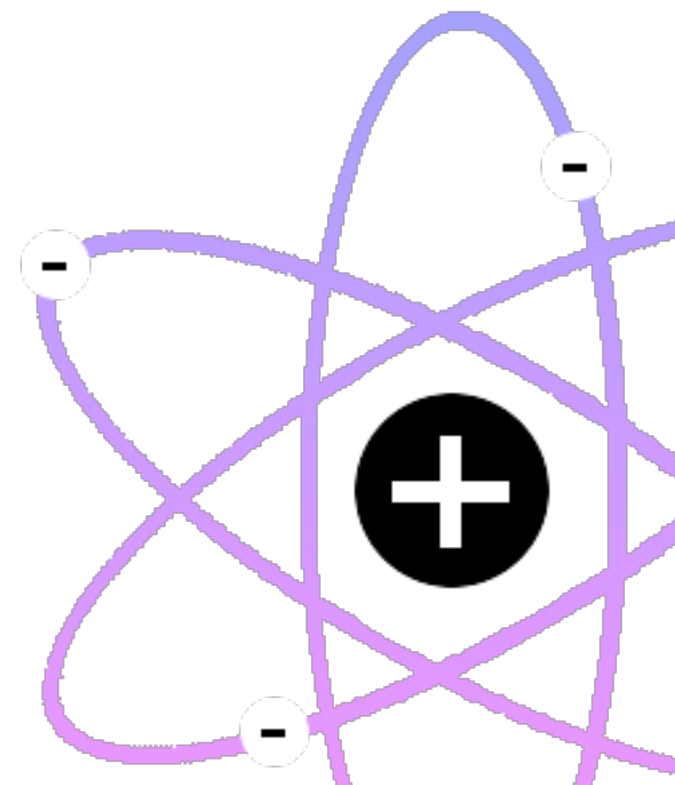
$$R_{n,l}(r) = \sqrt{\frac{2^3 (n-l-1)!}{na_0 2n [(n+l)!]^3}} e^{-r/na_0} \left(\frac{2r}{na_0}\right)^l L_{n-l-1}^{2l+1}\left(\frac{2r}{na_0}\right),$$

where $n \in \mathbb{N}$ & $n \geq l+1$

And let us not forget
the electron spin too!

$$j = l \pm \frac{1}{2}, \quad j > 0.$$

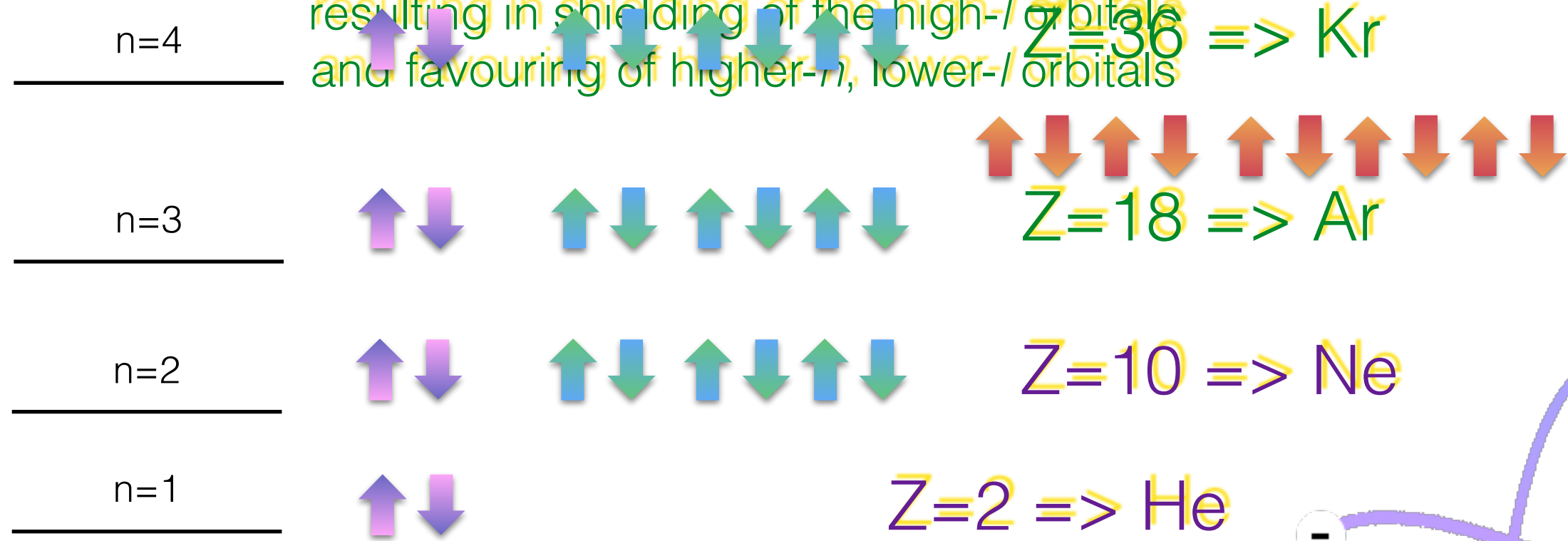
Quantisation is a natural outcome of
solving the Schrödinger equation



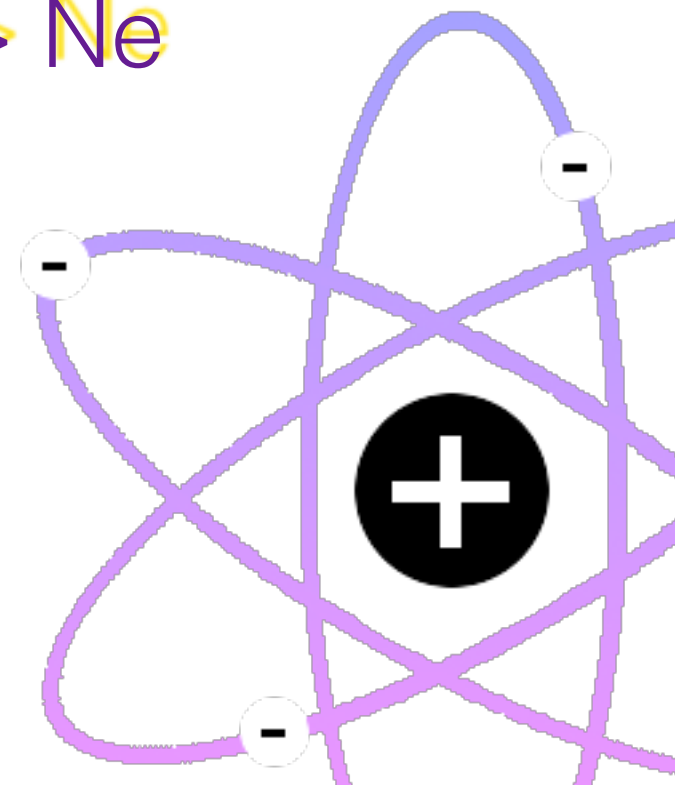
Building up the atom

Ordering up the levels reveals some known pattern

Atoms are not hydrogen-like and the electrons interact with one another resulting in shielding of the high- l orbitals and favouring of higher- n , lower- l orbitals



The natural appearance of atomic magic numbers

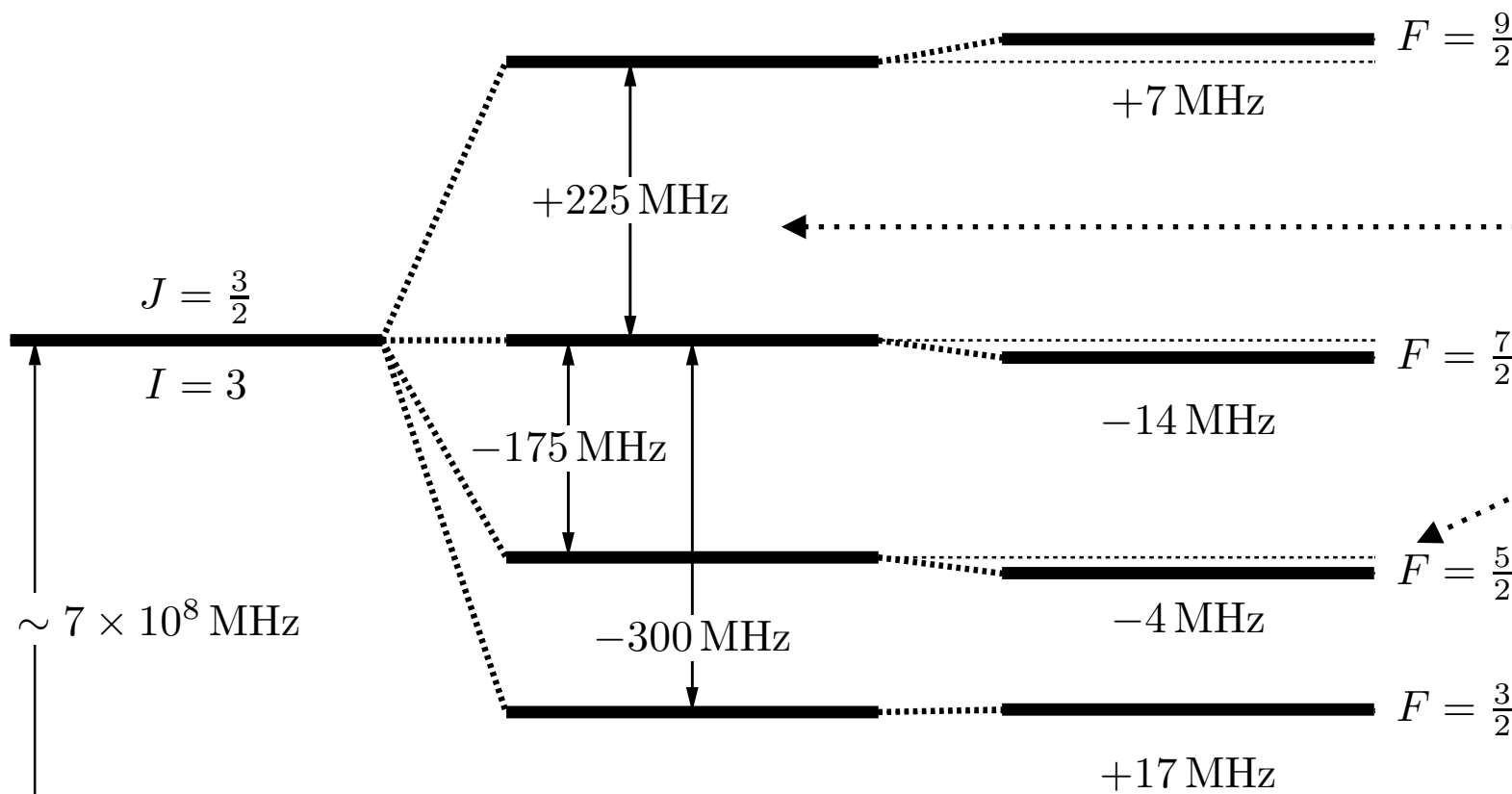


Taking a closer look

Hyperfine structure of the atomic levels

$$F = I + J,$$

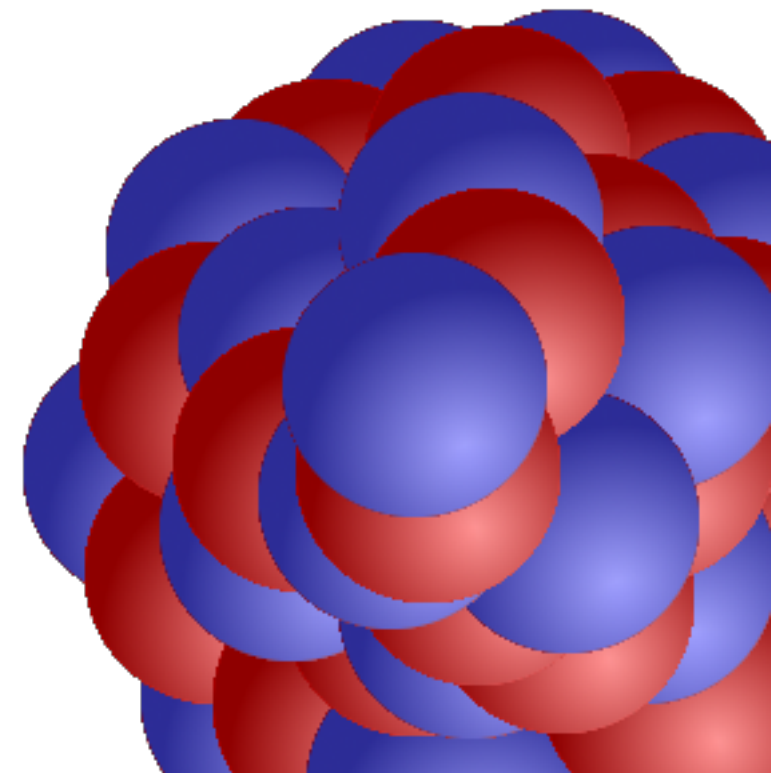
$$|I - J| \leq F \leq I + J.$$



$$\Delta E \sim \mu, Q$$

$$V_{Coulomb} + V_{Dipole} + V_{Quadrupole}$$

The nucleus is not a point charge!

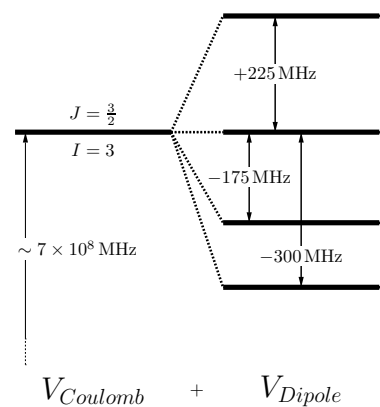


Hyperfine structure

Let's do the math!

$$\mathbf{F} = \mathbf{I} + \mathbf{J},$$

$$|I - J| \leq F \leq I + J.$$



$$\Delta E = \frac{A}{2} K$$

$$A = \frac{\mu B_0}{IJ}$$

$$K = F(F + 1) - I(I + 1) - J(J + 1)$$

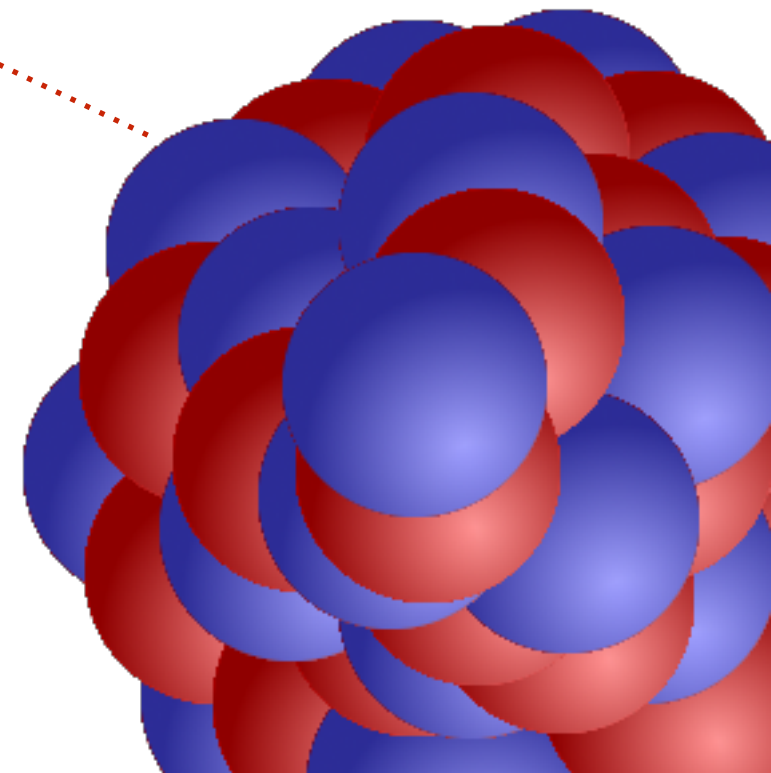
$$I, J > 0$$

Measuring the nuclear spin!

$$\frac{A_1}{A_2} = \frac{\mu B_{01}}{I J_1} \frac{I J_2}{\mu B_{02}} = \frac{B_{01} J_2}{B_{02} J_1},$$

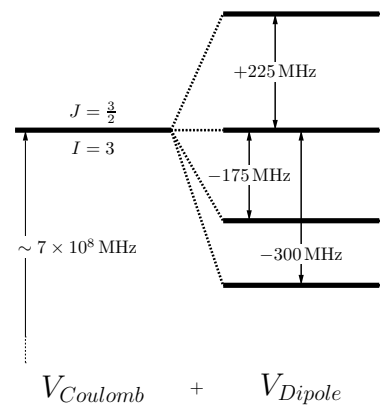
$$\frac{A}{A'} = \frac{\mu B_0}{I J} \frac{I' J}{\mu' B_0} = \frac{\mu I'}{\mu' I}$$

Magnetic dipole moment



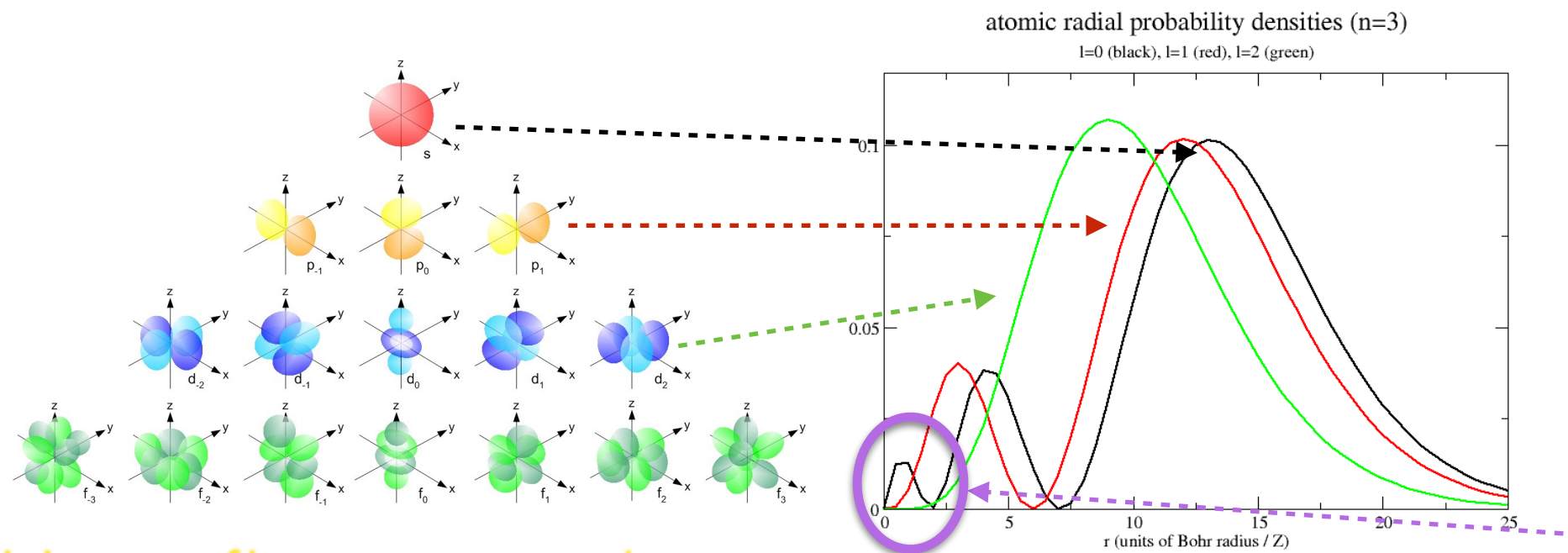
Hyperfine structure

A perturbation of a perturbation of a perturbation of a perturbation



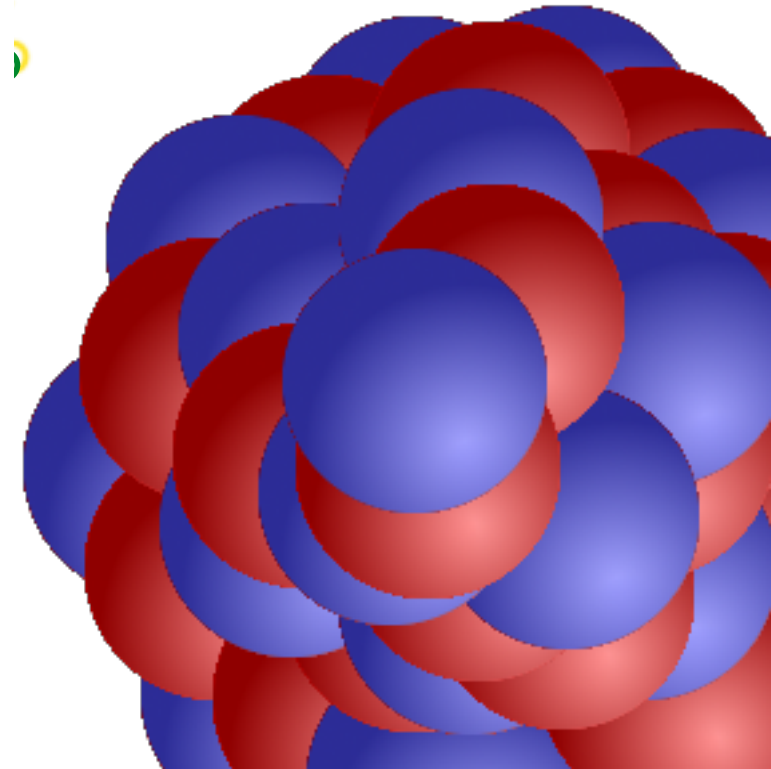
$$\Delta E = \frac{A}{2} K \quad A = \frac{\mu B_0}{IJ}$$

For s and $p_{1/2}$ orbitals, B_0 is not uniform over the nuclear volume
Averaging the interaction over the volume induces a correction



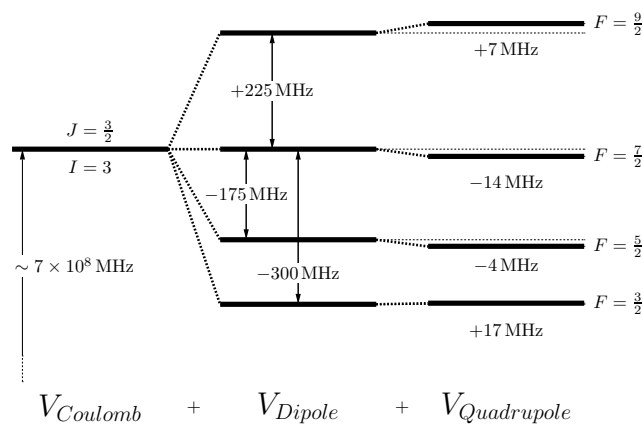
Hyperfine anomaly

$$\frac{A}{A'} = \frac{\mu I'}{\mu' I} \frac{1 + \epsilon}{1 + \epsilon'} \approx \frac{\mu I'}{\mu' I} (1 + \Delta')$$



Hyperfine structure

Let's do the math!



$$\mathbf{F} = \mathbf{I} + \mathbf{J},$$

$$|I - J| \leq F \leq I + J.$$

$$\Delta E = \frac{B}{2} \frac{3K(K+1) - 2I(I+1)2J(J+1)}{2I(2I-1)2J(2J-1)}$$

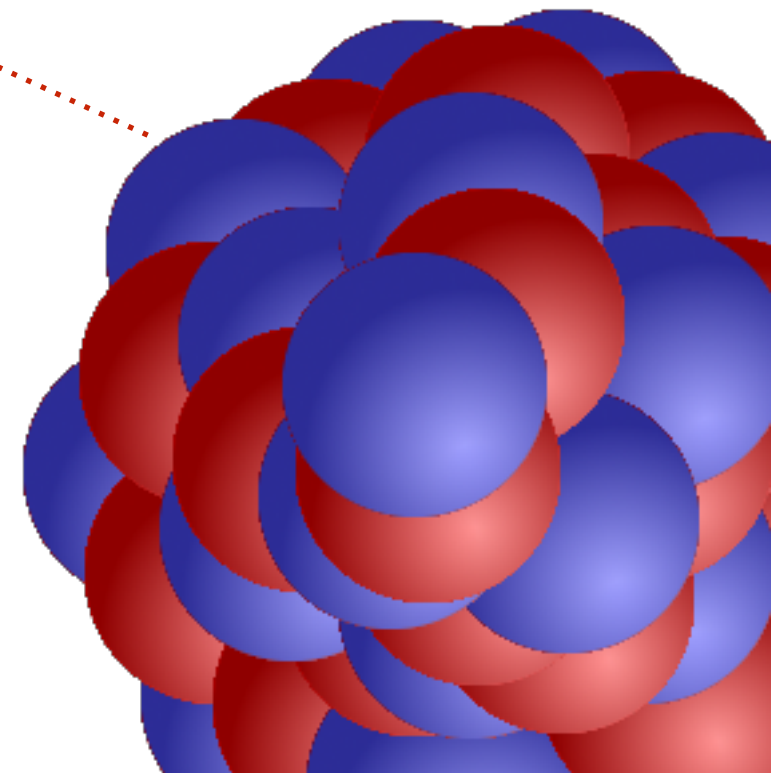
$$B = \frac{eQ}{4} \frac{\partial^2 V}{\partial z^2}$$

$$K = F(F+1) - I(I+1) - J(J+1)$$

$$I, J > 1/2$$

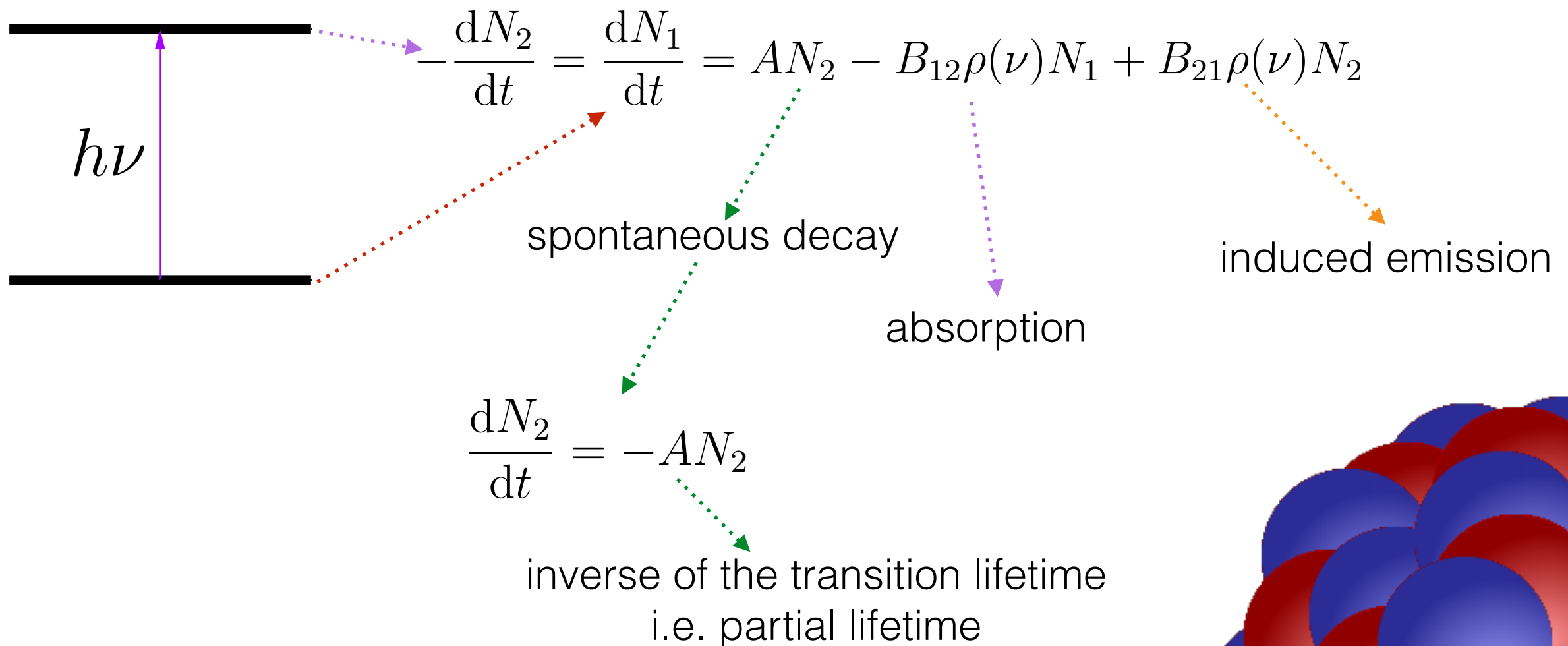
A perturbation
of a perturbation
of a perturbation
of a perturbation

Electric quadrupole moment



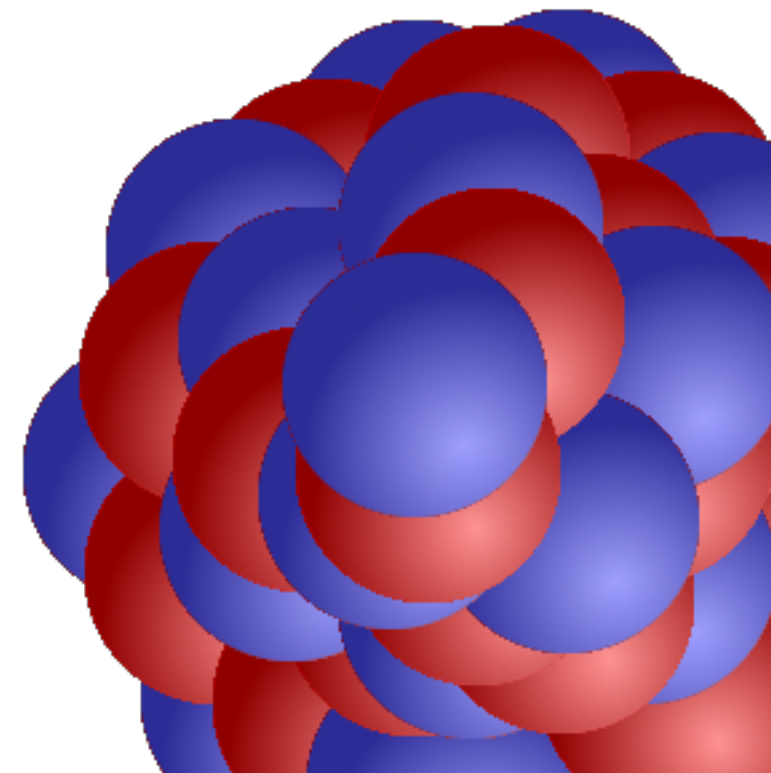
Atomic transitions

A question of intensity



Einstein coefficients

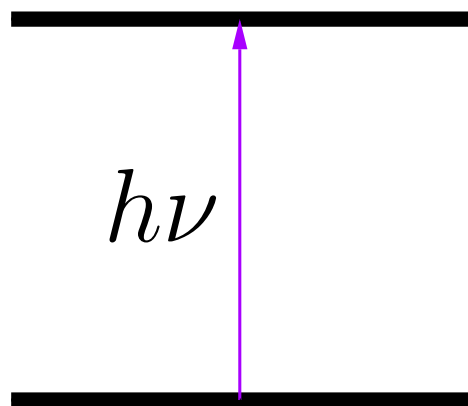
$$\tau = 1 / \sum_i A_i$$



Atomic transitions

What the atom may or may not do

To first order, the photon field can be considered as an electric dipole field (E1)



$$\begin{aligned} \Delta l &= \pm 1 \\ \Delta J &= 0, \pm 1, \quad J = 0 \not\rightarrow 0 \\ \Delta F &= 0, \pm 1, \quad F = 0 \not\rightarrow 0 \end{aligned}$$

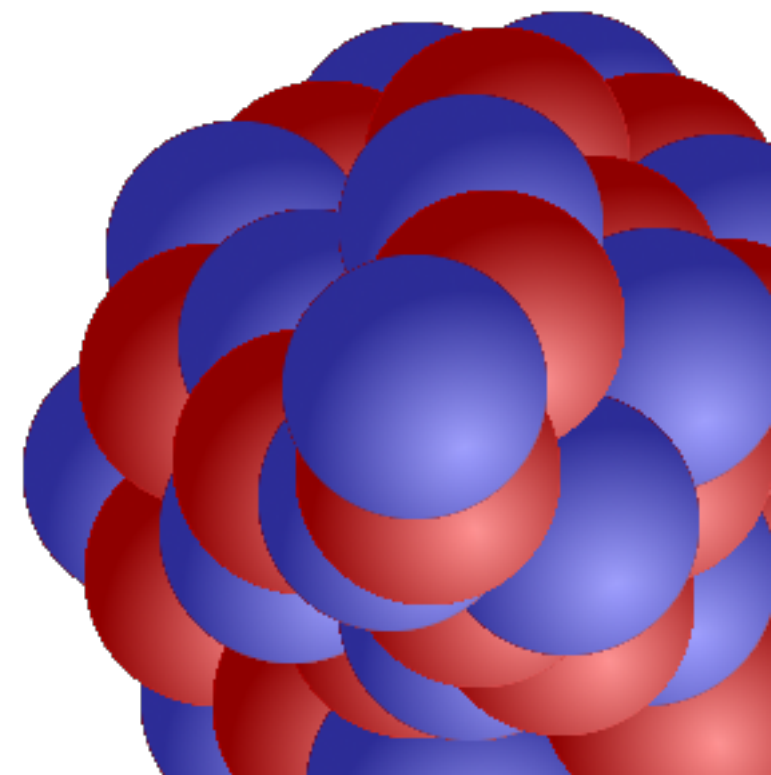
Parity change

1 unit of angular momentum

carried by the photon
=> triangular relation

Selection rules

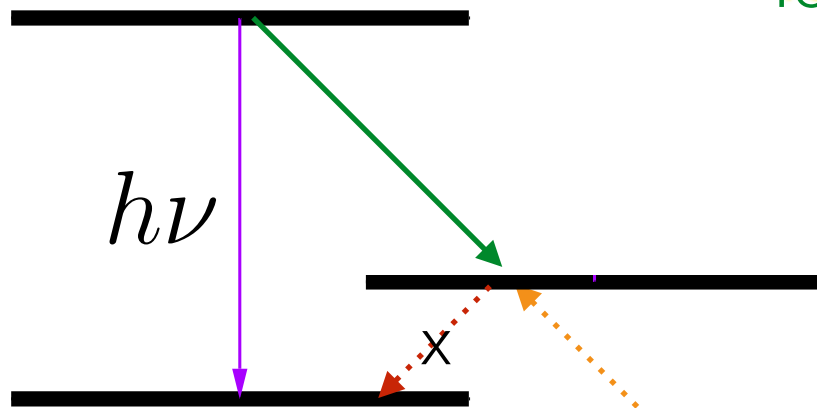
$$s \leftrightarrow p, p \leftrightarrow d, d \leftrightarrow f, \dots$$



Atomic transitions

What the atom may or may not do

To first order, the photon field can be considered as an electric dipole field (E1)

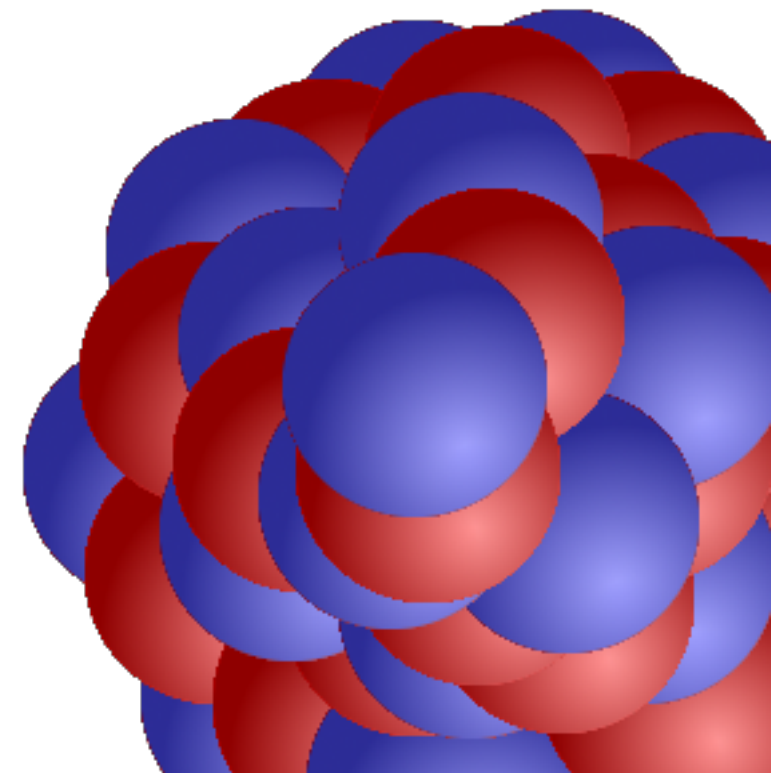


$$\begin{aligned}\Delta l &= \pm 1 \\ \Delta J &= 0, \pm 1, \quad J = 0 \not\rightarrow 0 \\ \Delta F &= 0, \pm 1, \quad F = 0 \not\rightarrow 0\end{aligned}$$

Low-energy atomic levels which cannot satisfy these rules towards the atomic ground state will trap electrons with no (easy) means of decay.

Metastable states

$$s \leftrightarrow p, p \leftrightarrow d, d \leftrightarrow f, \dots$$



Atomic transitions

More math?!

$$\langle F_f, m_f | \mathbf{e} \cdot \mathbf{d} | F_i, m_i \rangle$$

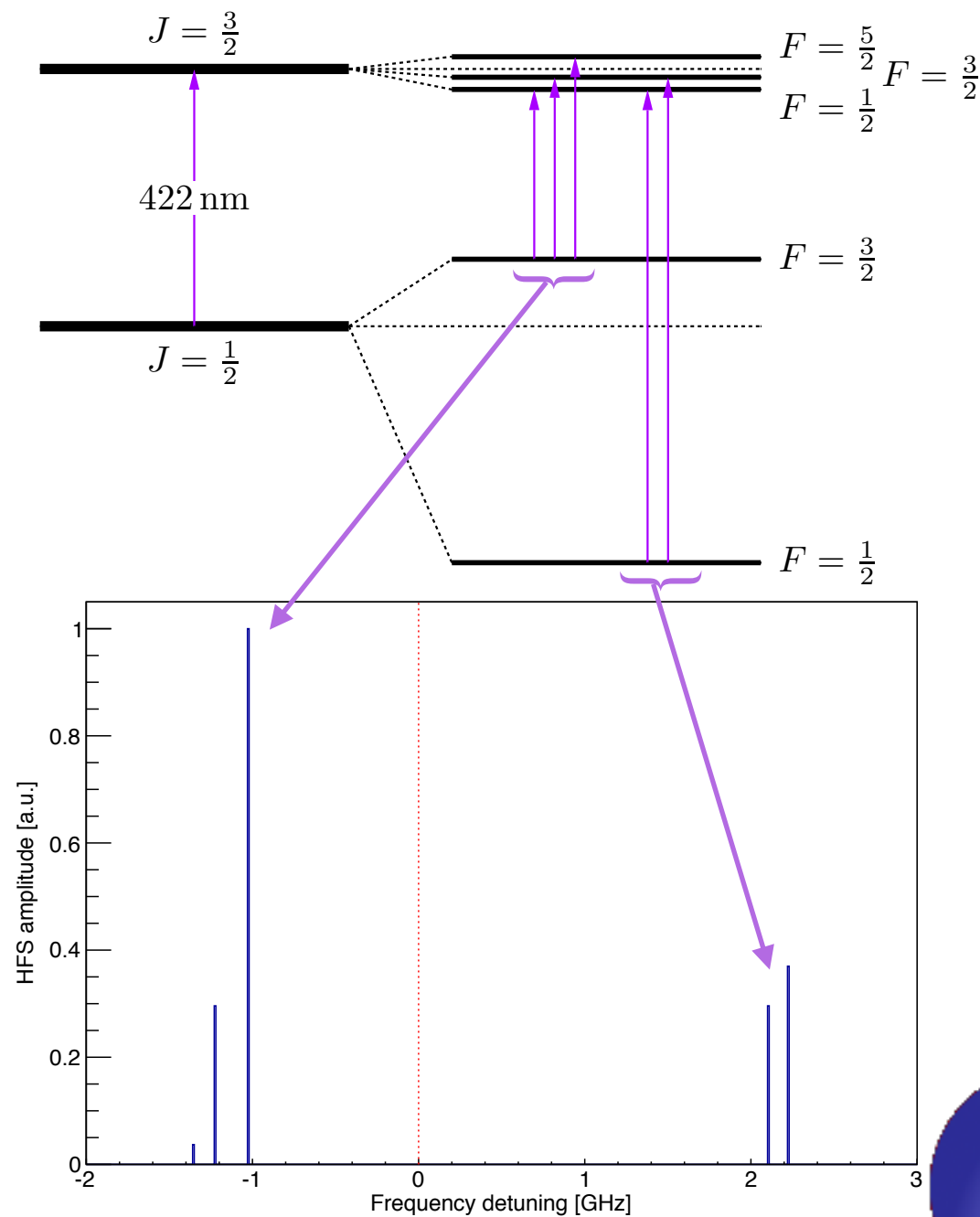
$$(2F_i + 1)(2F_j + 1) \times$$

$$\left(\begin{array}{ccc} F_f & 1 & F_i \\ -m_f & 0 & m_i \end{array} \right)^2 \left\{ \begin{array}{ccc} J_f & F_f & I \\ F_i & J_i & 1 \end{array} \right\}^2$$

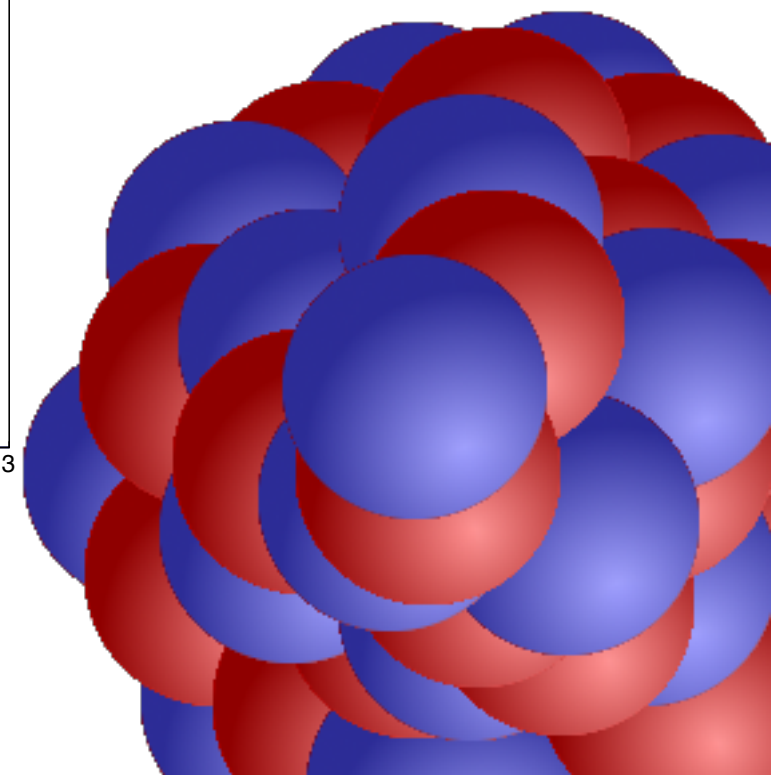
Selection rules (i.e. number of transitions) and relative amplitudes are spin dependent and can be used to determine I

Hyperfine transitions

4 ways of measuring the nuclear spin!



Intensities are proportional to the overlap of the final and initial quantum states under the action of the electric dipole operator



Atomic transitions

More perturbations

$$\delta\nu^{AA'} = \frac{A' - A}{AA'} \left(m_e \nu + M_{SMS} \right) + F \delta \langle r^2 \rangle^{AA'}$$

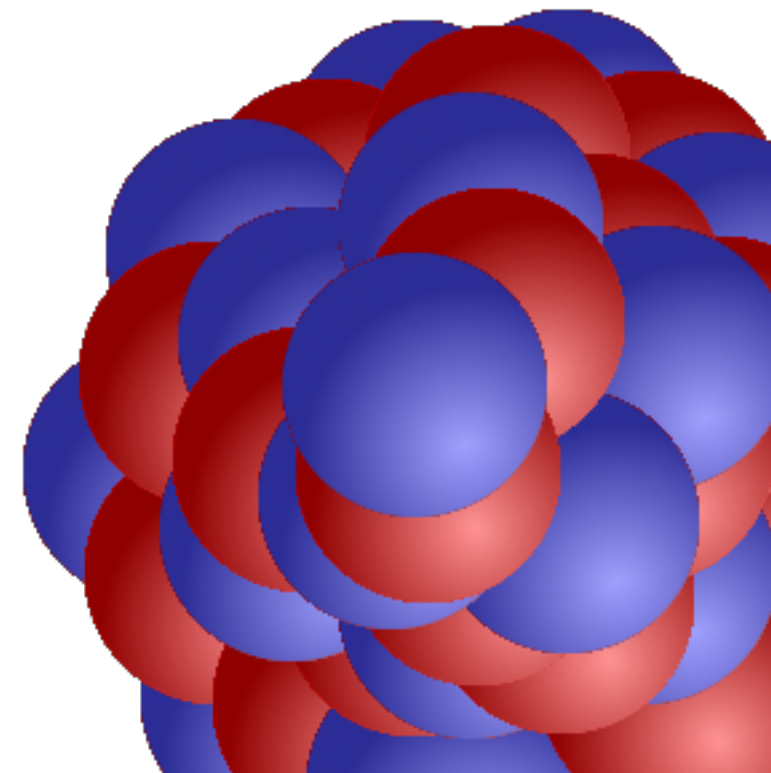
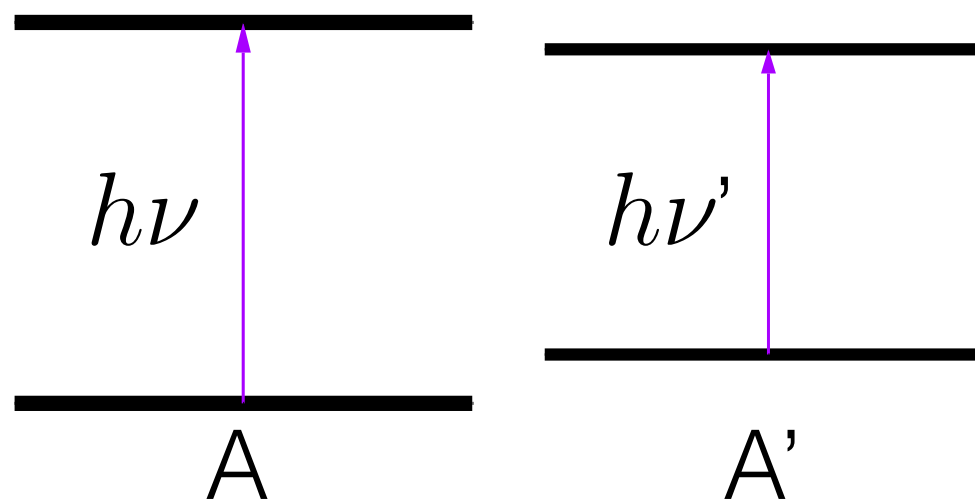
Mass shift

From one isotope to the next, the reduced mass of the nucleus + electron system varies, giving rise to a mass shift, scaling with A^{-2}

Field shift

From one nuclear state to the next, the charge distribution within the nucleus may vary, perturbing electron orbitals with a non-vanishing overlap with the nucleus

Isotope shift



Isotope shift

More details

$$\delta\nu^{AA'} = \frac{A' - A}{AA'} (m_e\nu + M_{SMS})$$

scales with A^{-2}

direct impact

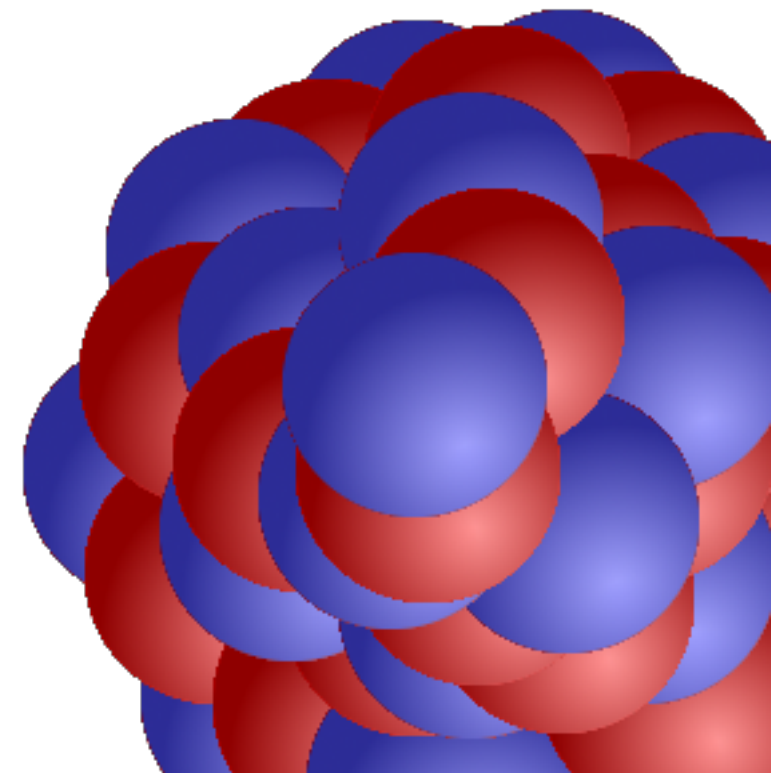
can easily be
accounted for

indirect impact

- ▶ negligible between nuclear isomers
- ▶ reduced impact in heavy systems

- ▶ arises from the rearranging of the electronic cloud
- ▶ cannot be analytically determined for more than 3 electrons
- ▶ must rely on large-scale calculations
- ▶ can be >0 or <0

Mass shift



Isotope shift

More details

M_{SMS} & F are
transition specific !

$$\delta\nu = F\delta\langle r^2 \rangle$$

scales with Z

no analytical
solution

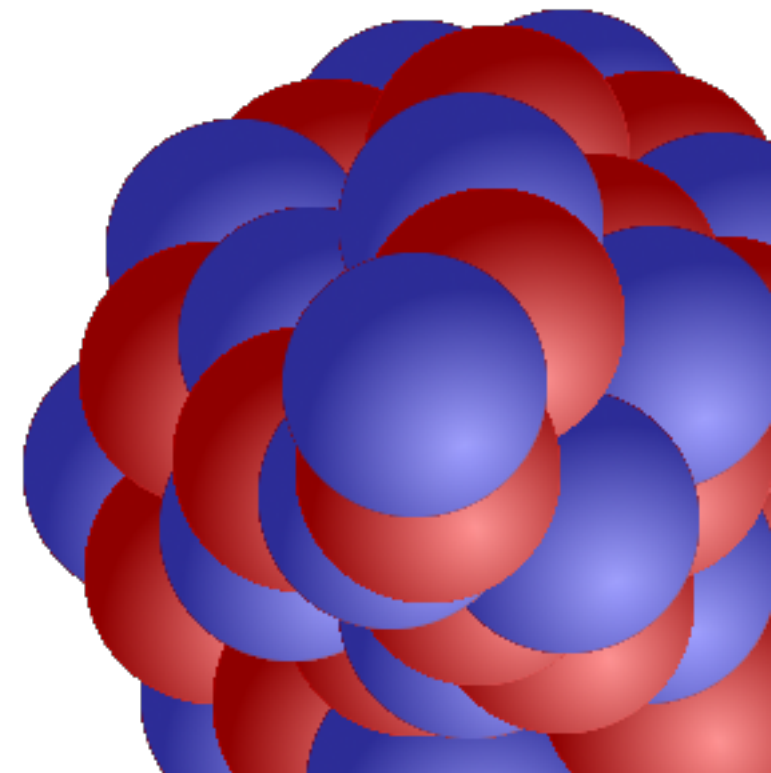
higher moments, e.g. $\delta\langle r^4 \rangle$
can be accounted for
directly

- ▶ sole contribution between nuclear isomers
- ▶ increased impact in heavy systems

- ▶ must rely on large-scale calculations for more than 3 electrons
- ▶ can be >0 or <0

Field shift

a.k.a. Volume shift



Isotope shift

$$\mu_{AA'} = \frac{AA'}{A'-A}$$

Experimental approach to avoid calculations

$$\mu_{AA'} \delta \nu^{AA'} = M + F \mu_{AA'} \delta \langle r^2 \rangle^{AA'}$$

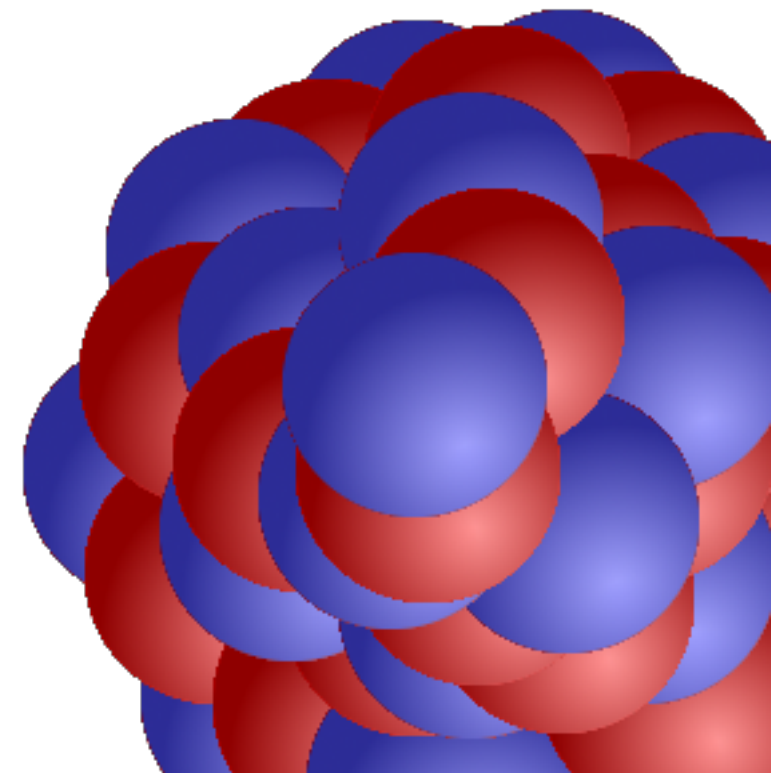
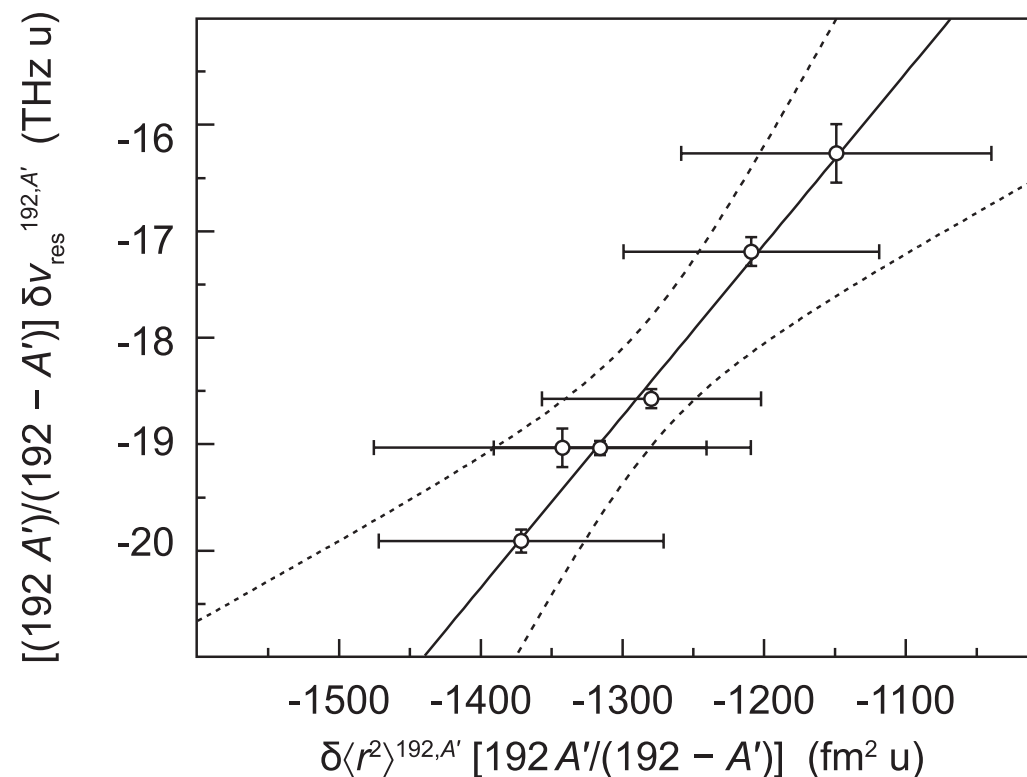
Experimental input
from laser / optical
spectroscopy

y intercept

slope

Experimental input from
electron scattering,
muonic decay, *K* x-ray
spectroscopy

King plot



Isotope shift

$$\mu_{AA'} = \frac{AA'}{A'-A}$$

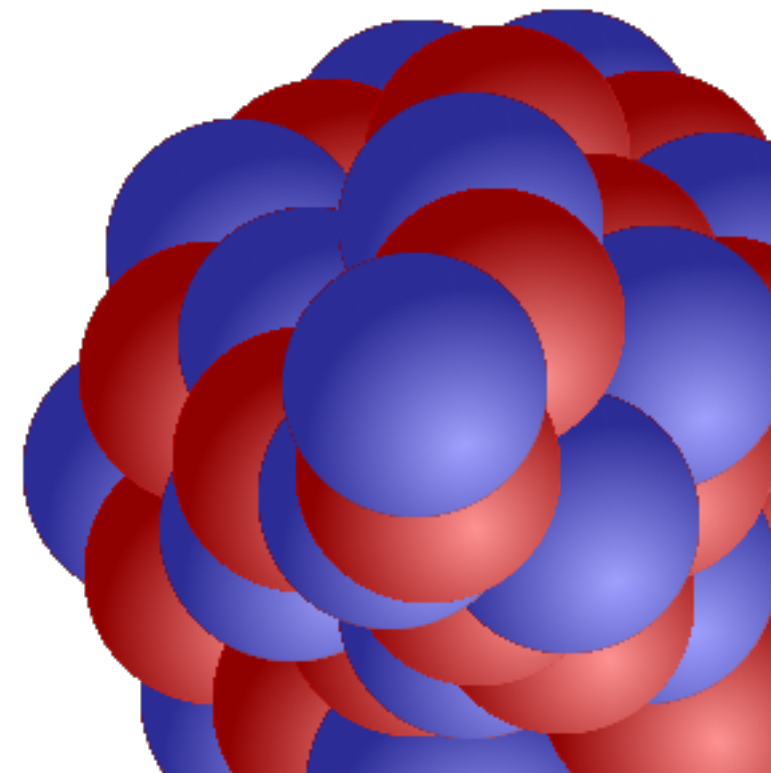
The art of fitting a straight line

$$\mu_{AA'} \delta \nu^{AA'} = M + F \mu_{AA'} \delta \langle r^2 \rangle^{AA'}$$

- 2 points => fitting a straight line
- 1 point => reference isotope
- TOTAL => 3 data points minimum

H	alkali &																			He
Li	Be	alkali-earth												B	C	N	O	F	Ne	
Na	Mg													Al	Si	P	S	Cl	Ar	
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr			
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe			
Cs	Ba	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn			
Fr	Ra	Lr	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cp									
		La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb					
		Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No					

King plot



Isotope shift

$$\mu_{AA'} = \frac{AA'}{A'-A}$$

Relating and testing

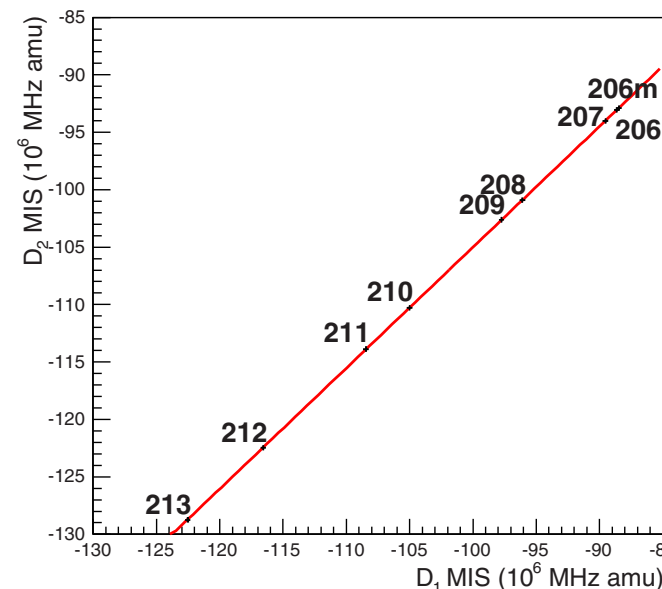
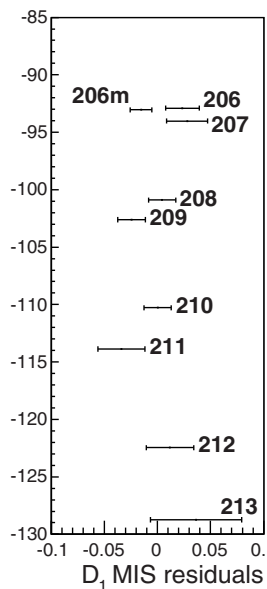
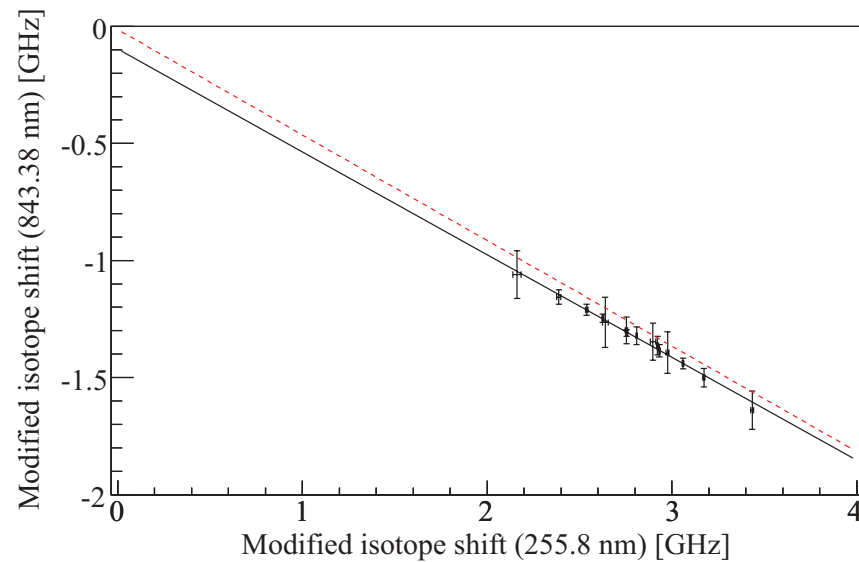
$$\mu_{AA'} \delta\nu_2^{AA'} = \frac{F_2}{F_1} \mu_{AA'} \delta\nu_1^{AA'} + \left(M_2 - \frac{F_2}{F_1} M_1 \right)$$

Transition 2

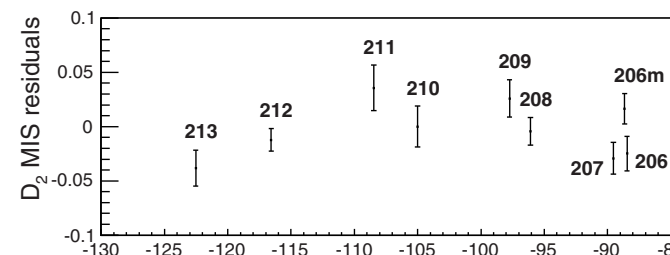
slope

Transition 1

y intercept relates the mass shifts to one another

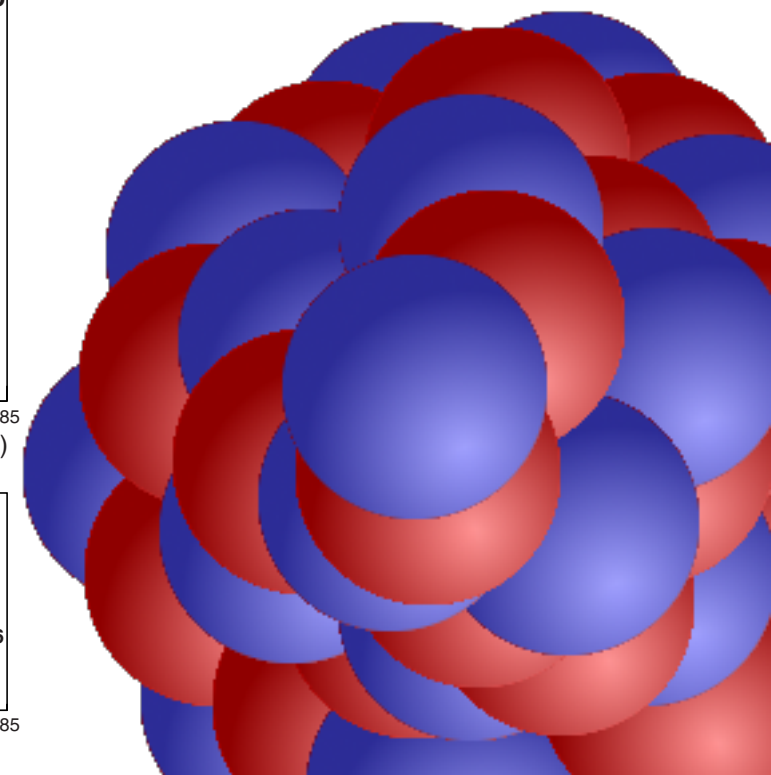


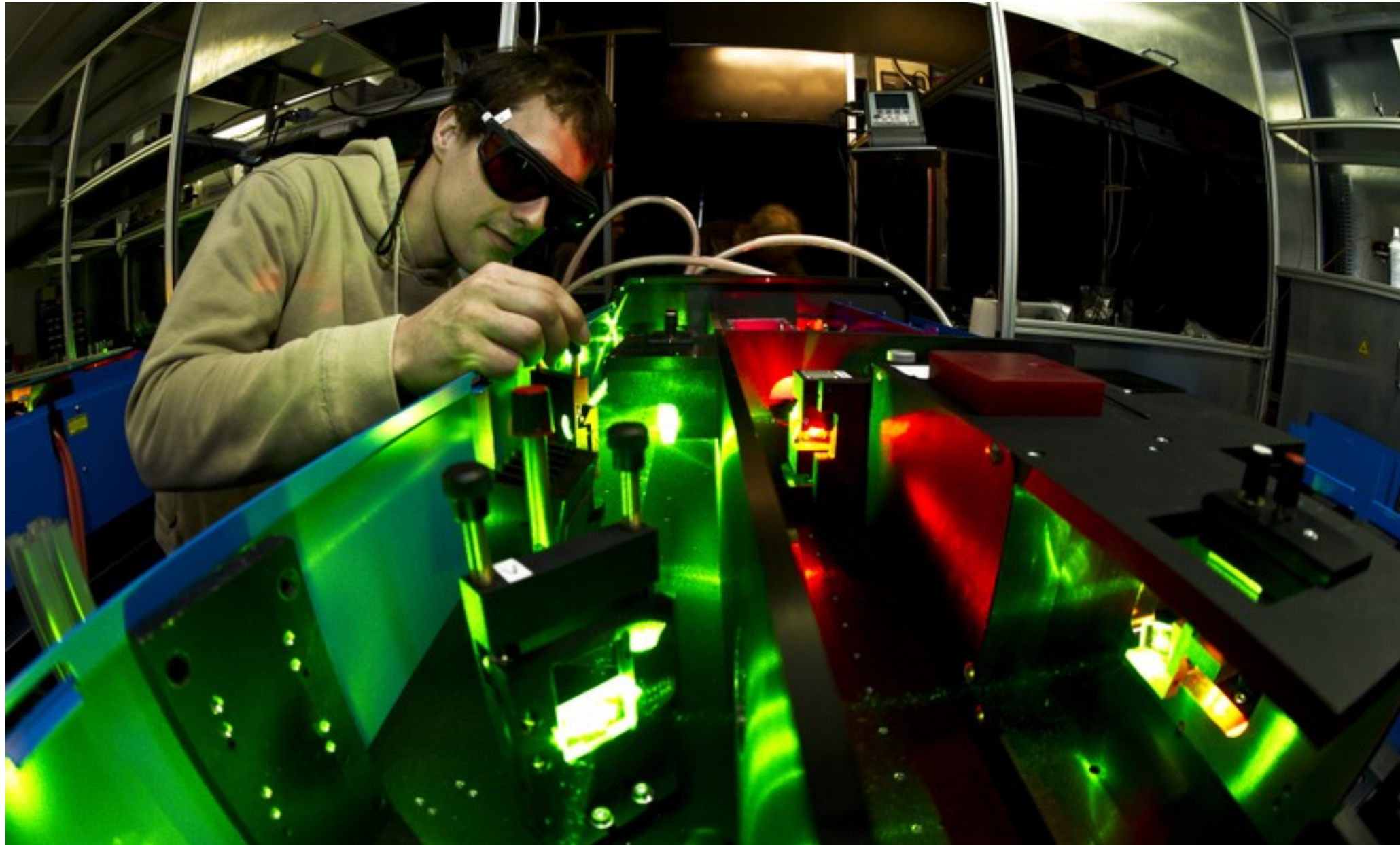
Fit Results	
χ^2 / ndf	$= 7.00094 / 7$
slope	$= 1.0521 \pm 0.0008$
int	$= 194 \pm 78 \text{ GHz amu}$



Modified King plot

Many more data available





RIB Production

Isotope production

Start with your favourite facility

fragmentation — ISOL —

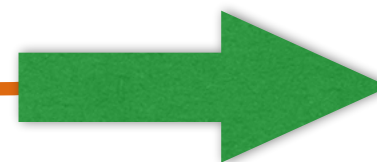
light projectile



thick target



hot line



ion source

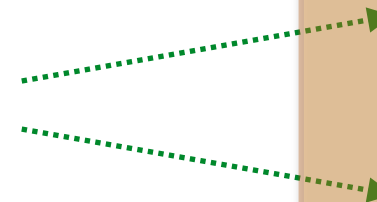
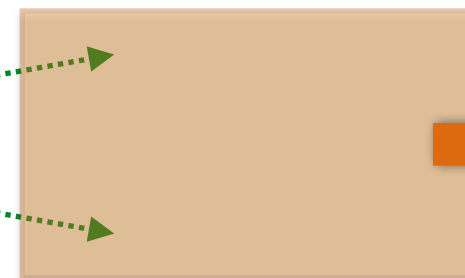
heavy projectile



thin target



gas catcher



recoils come out

Beam properties

ISOL

- Thick target
 - ▶ no refractory elements
 - ▶ slow (>ms) extraction
- Mass separation
 - ▶ high purity
- Bespoke ion sources
 - ▶ surface source for low IP
 - ▶ plasma for gases
 - ➔ non discriminate

fragmentation

- Thin target
 - ▶ all elements
 - ▶ fast extraction
- Pre mass separation
 - ▶ low purity
- Gas catcher
 - ▶ slow thermalisation (>ms)
 - ▶ survival
 - ➔ non discriminate

Beam properties

ISOL

- Thick target
 - ▶ no refractory elements
 - ▶ slow (>ms) extraction
- Mass separation
 - ▶ high purity
- **Bespoke ion sources**
 - ▶ surface source for low IP
 - ▶ plasma for gases
 - ➔ non discriminate

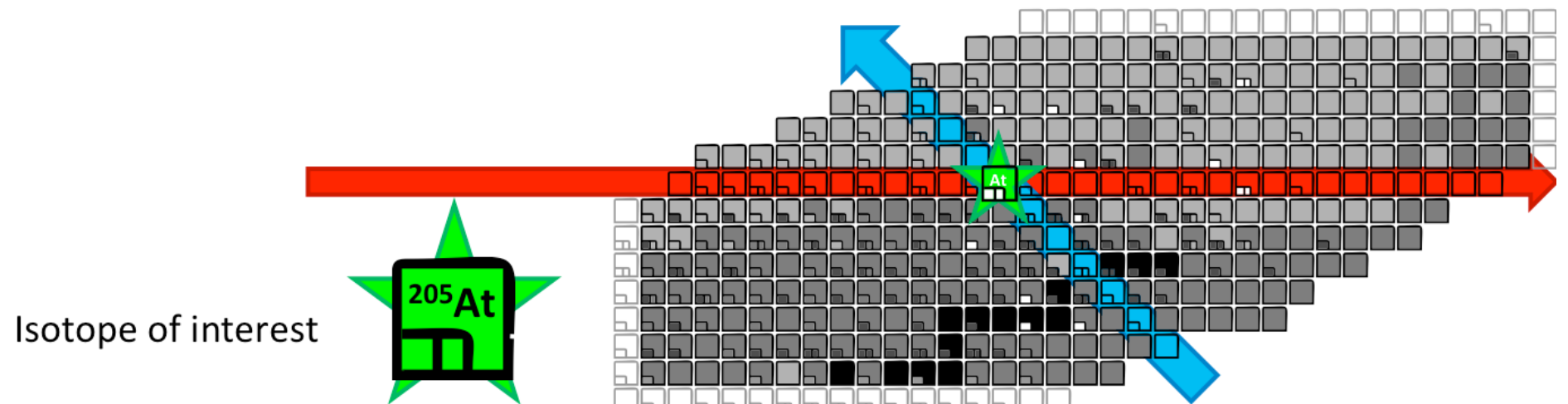
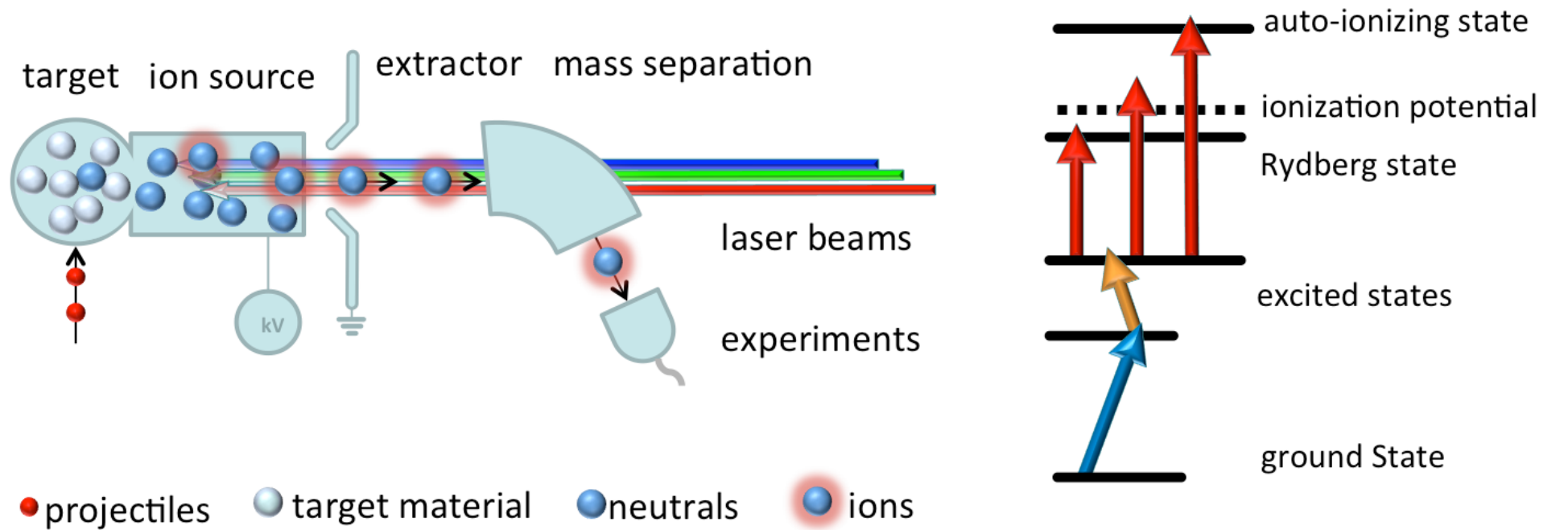
fragmentation

- Thin target
 - ▶ all elements
 - ▶ fast extraction
- Pre mass separation
 - ▶ low purity
- **Gas catcher**
 - ▶ slow thermalisation (>ms)
 - ▶ survival
 - ➔ non discriminate

RILIS

Resonance Ionisation Laser Ion Source

- ▶ Resonant transitions specific to a single element are chosen
- ▶ No single transition has enough energy to non-resonantly ionise an atom directly

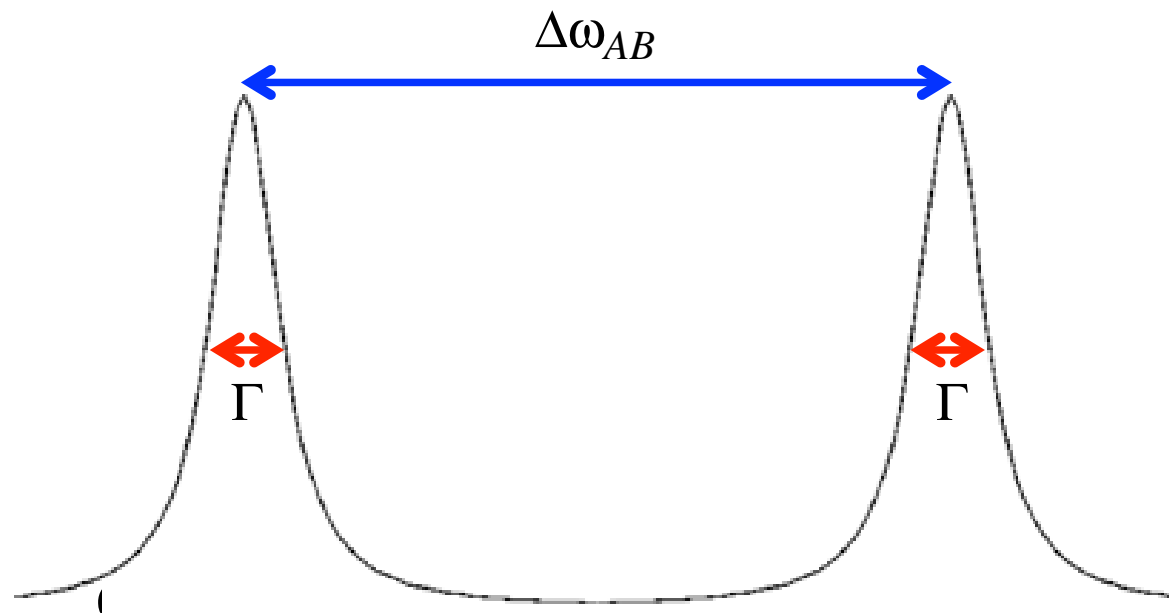


Everybody's favourite ion source

Selectivity

The power of a pure beam

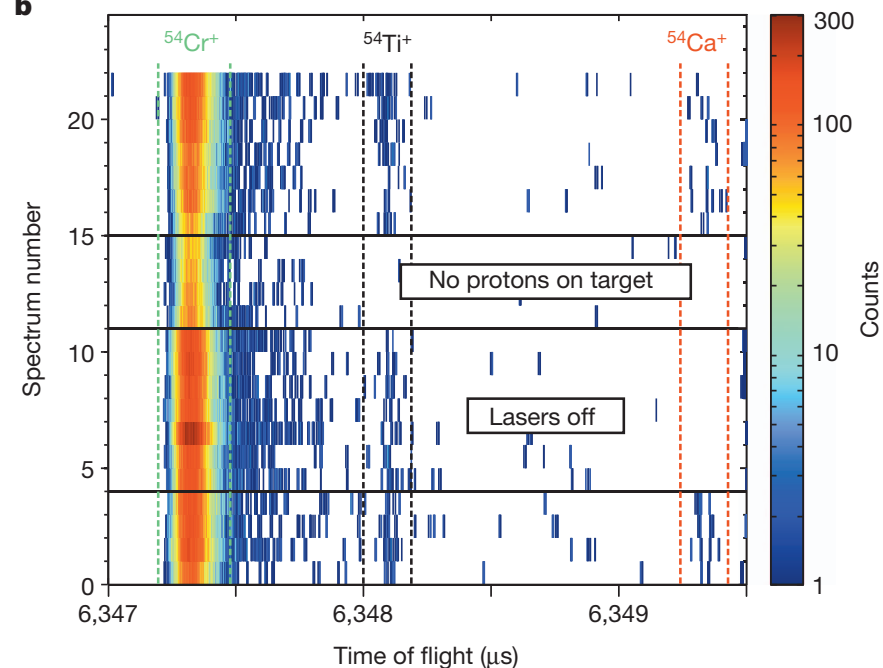
$$S = \left(\frac{\Delta\omega_{AB}}{\Gamma} \right)^2$$



- ▶ Each resonant transition has an associated selectivity.
- ▶ The selectivity of different transitions multiplies over.

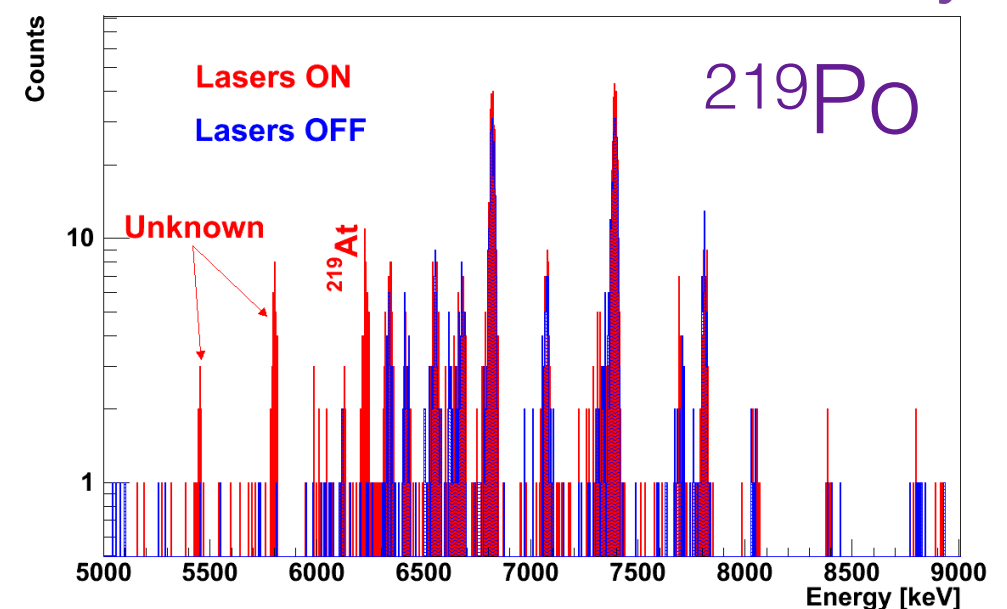
Laser ON/OFF

in mass measurement



in a decay

Si2 - A=219 - Lasers ON vs. OFF



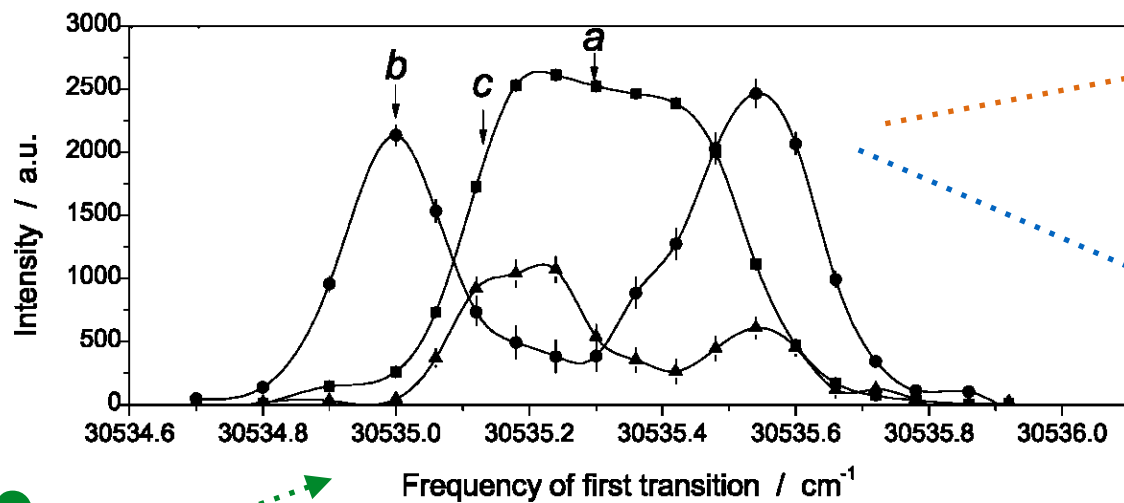
Isomeric beams

Different J^{π} means different HFS
=> enhanced selectivity

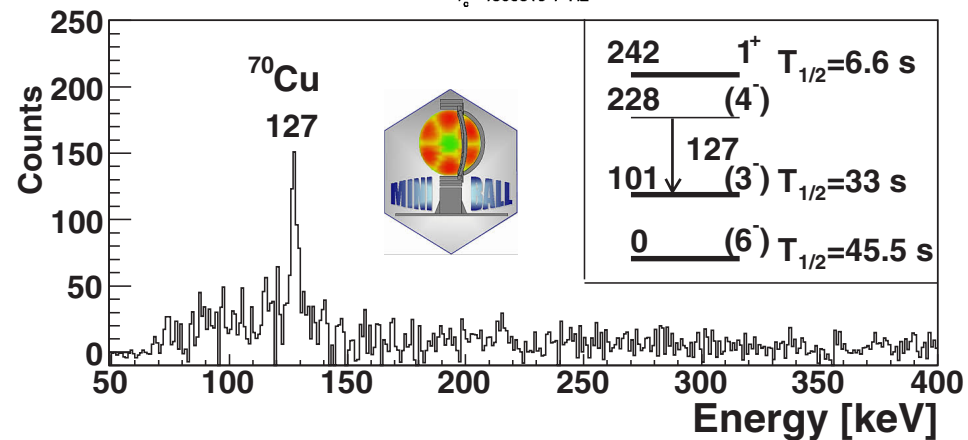
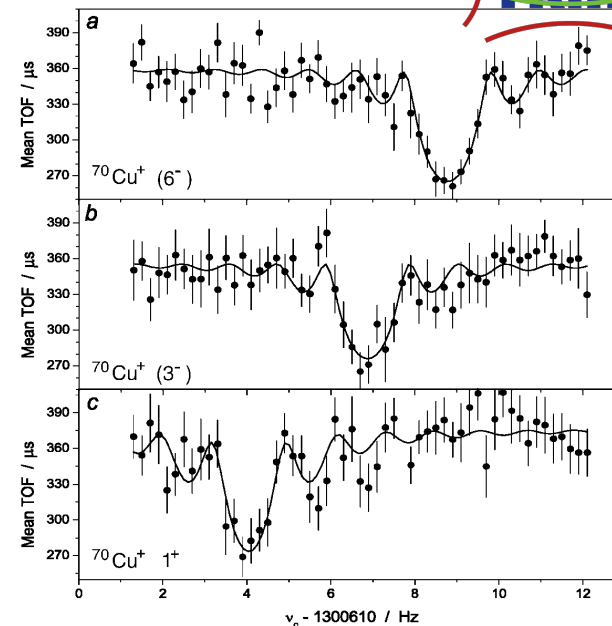
ISOL
TRAP

in mass measurement

in Coulomb excitation

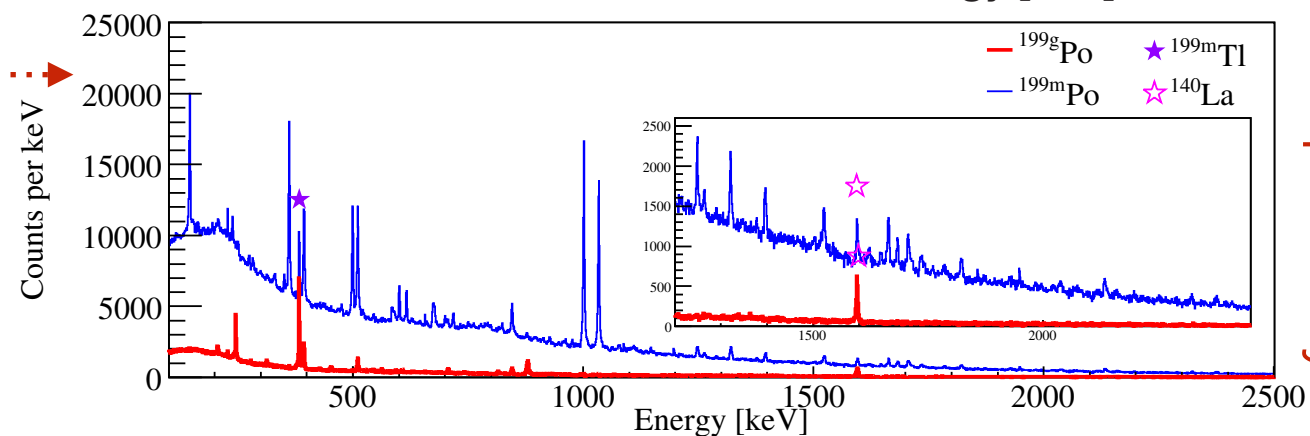
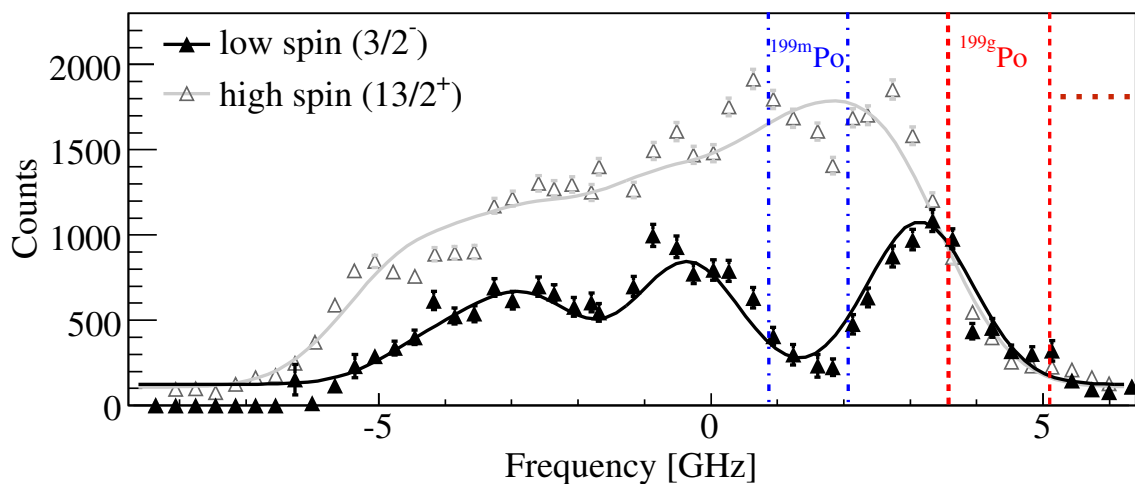


^{70}Cu



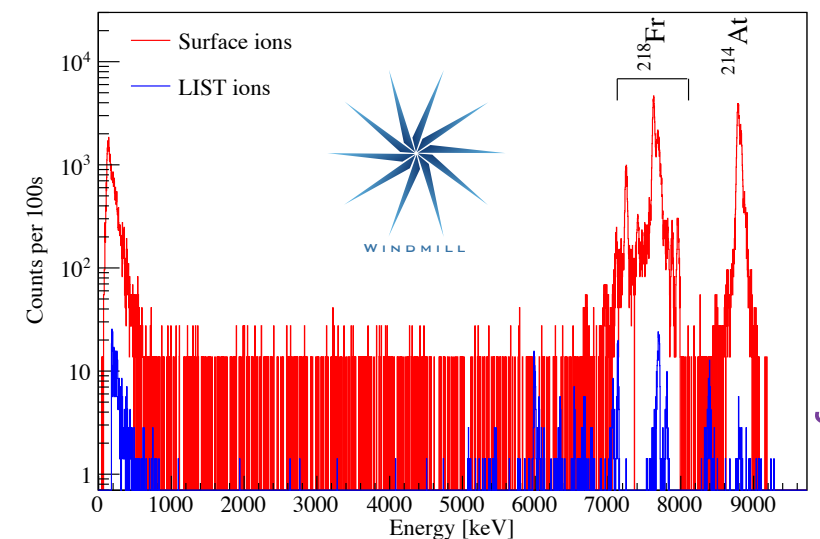
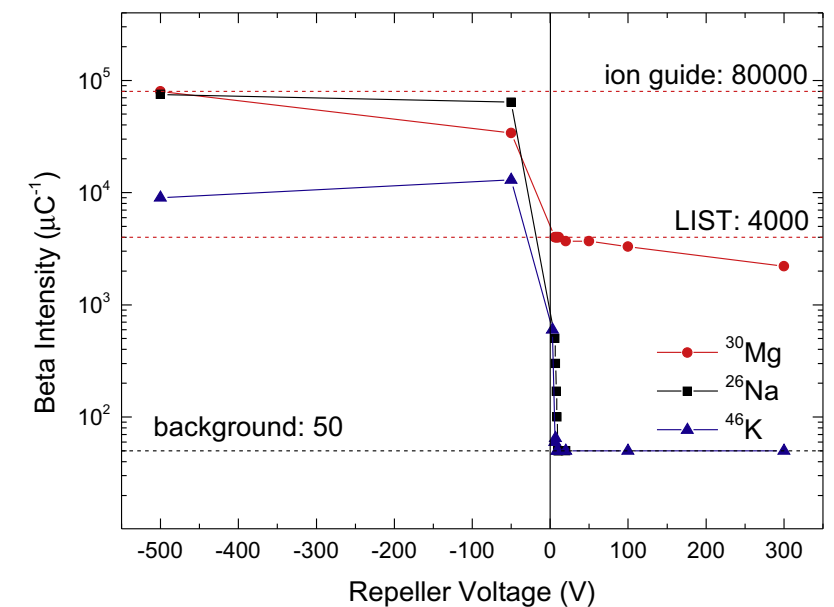
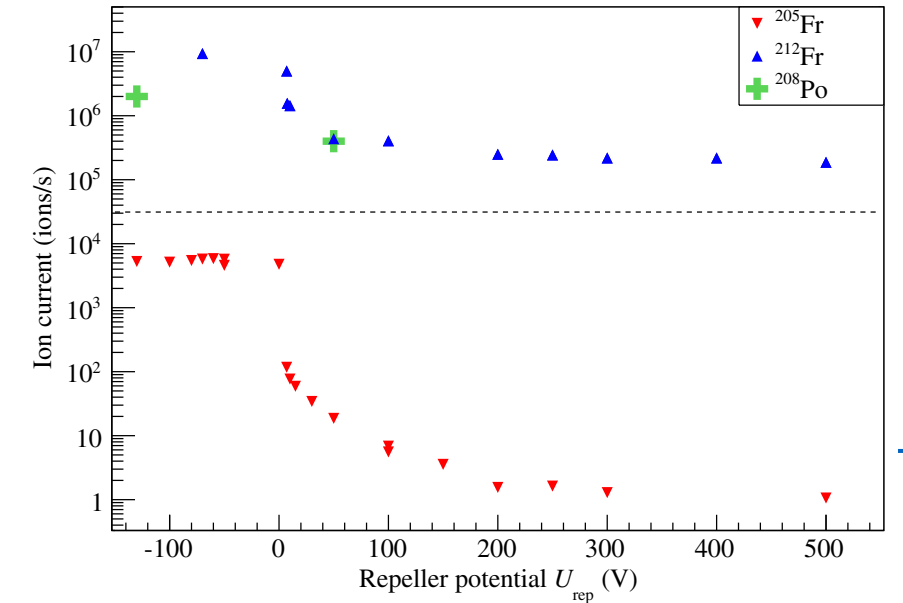
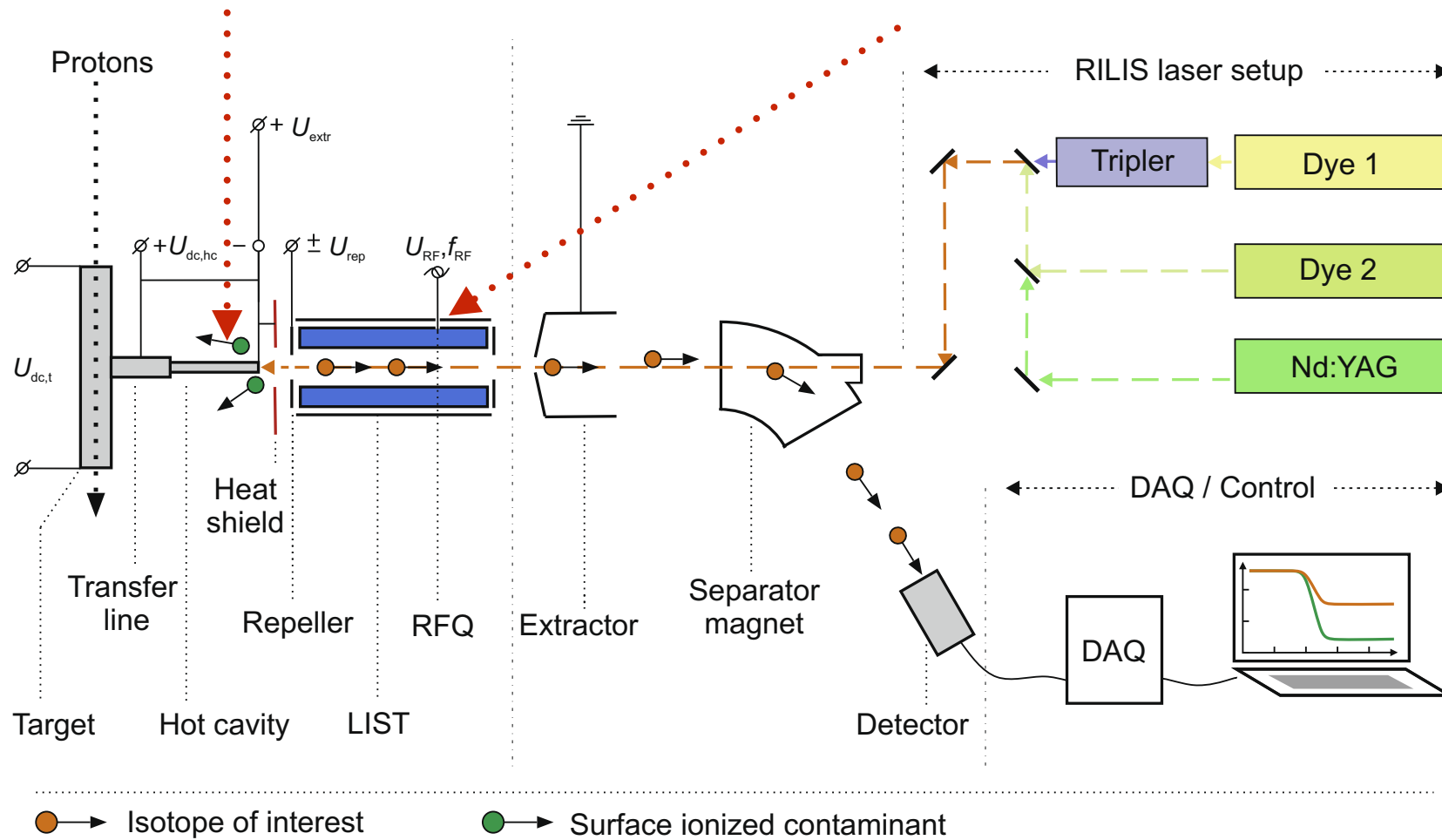
RILIS

^{199}Po



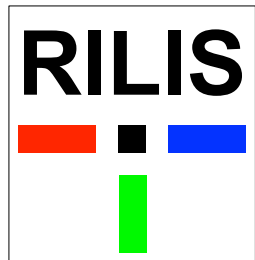
in β decay

Overcoming contaminants by geometrically decoupling the **atomiser** from the **ionising volume**

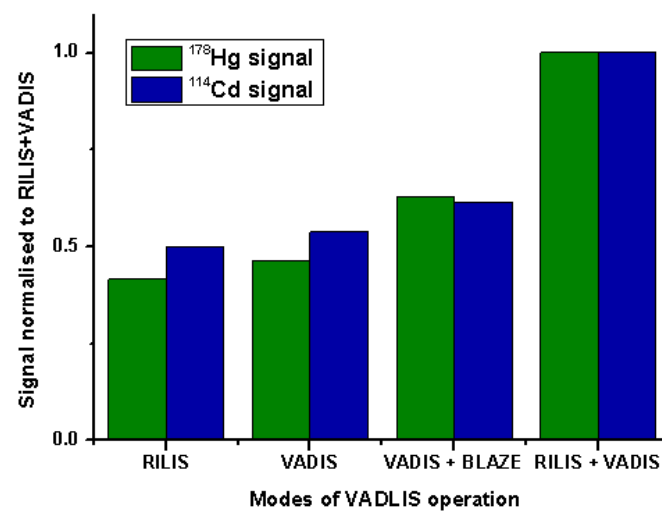
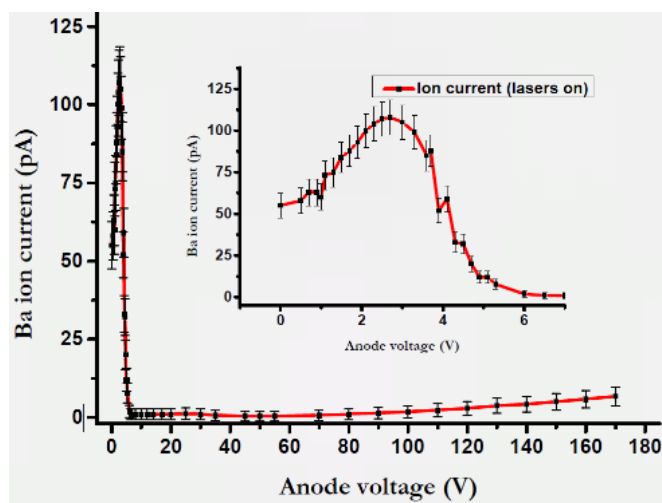
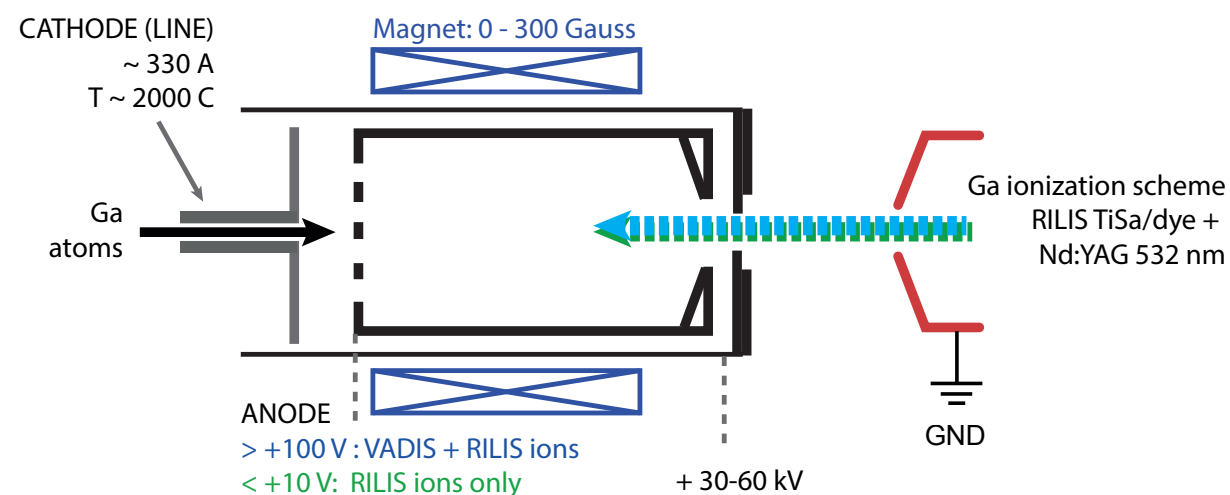
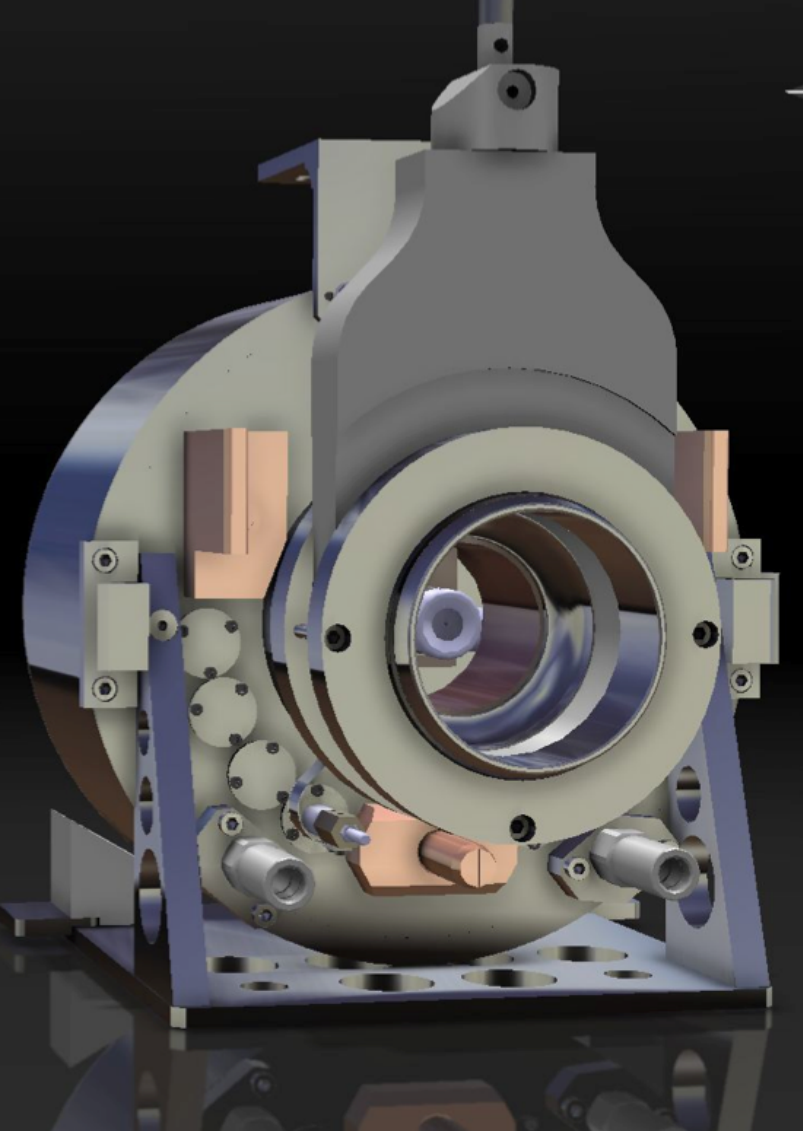


repeller scans

in a decay



VADLIS

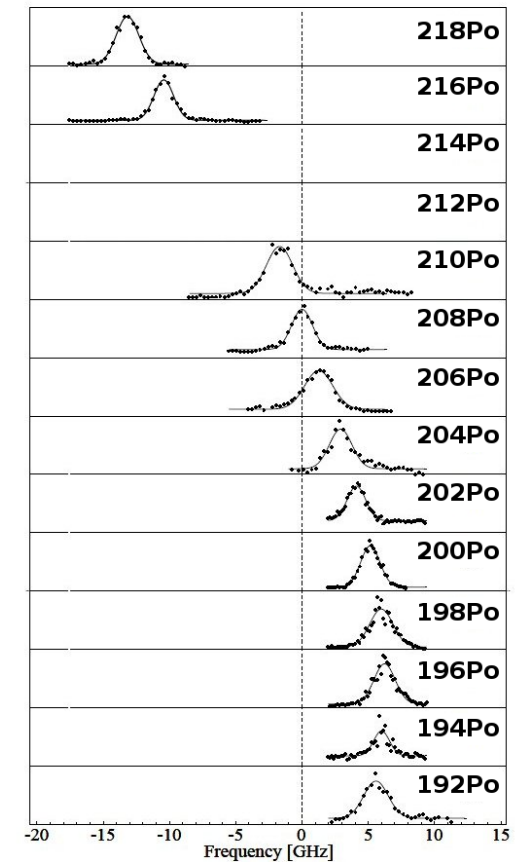
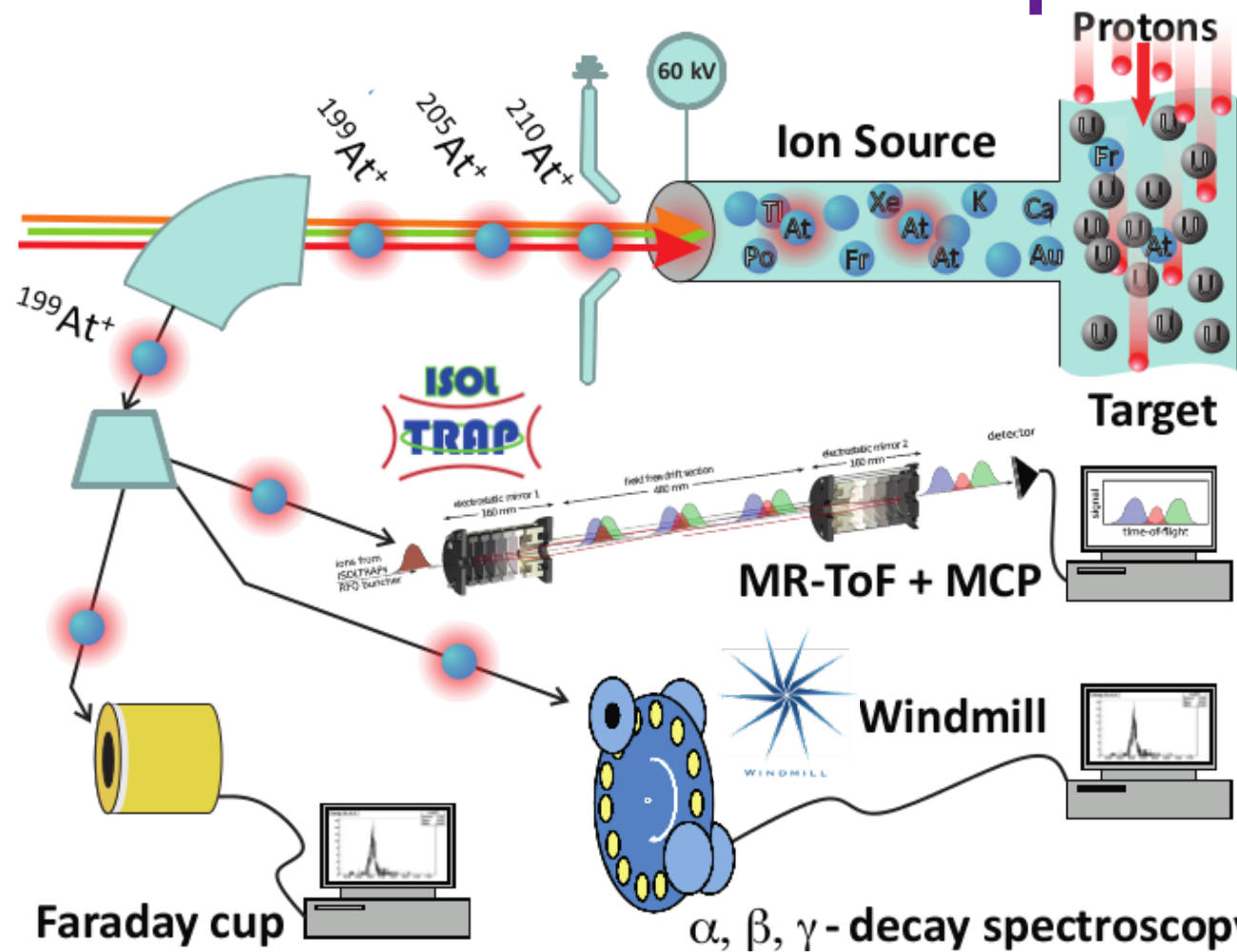


Versatility in the use of the VADIS with laser

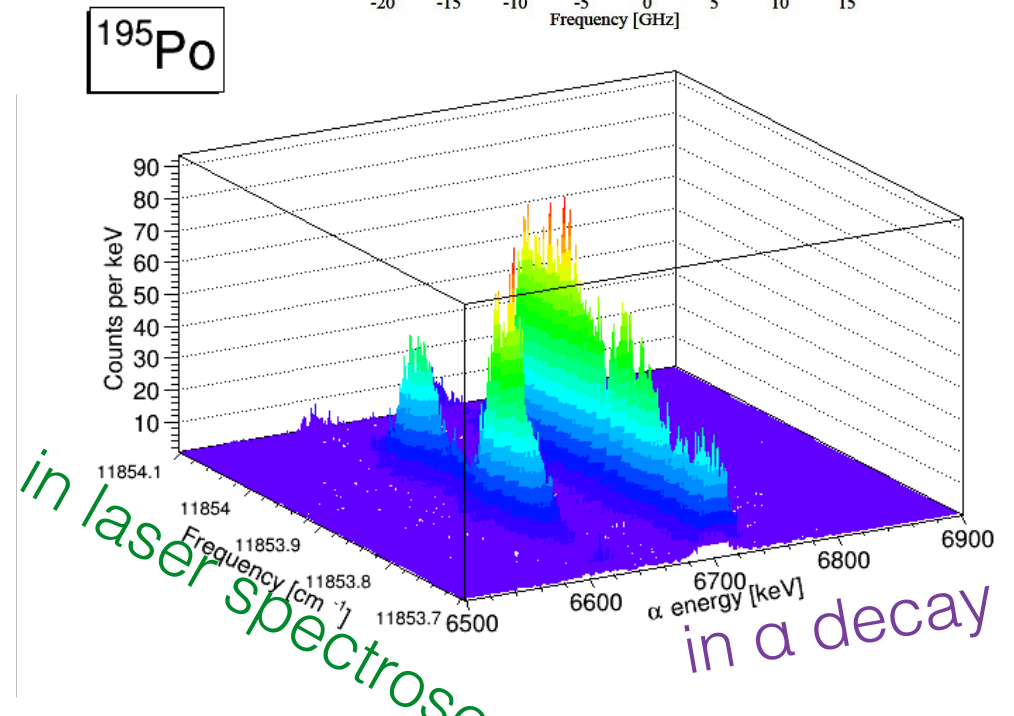
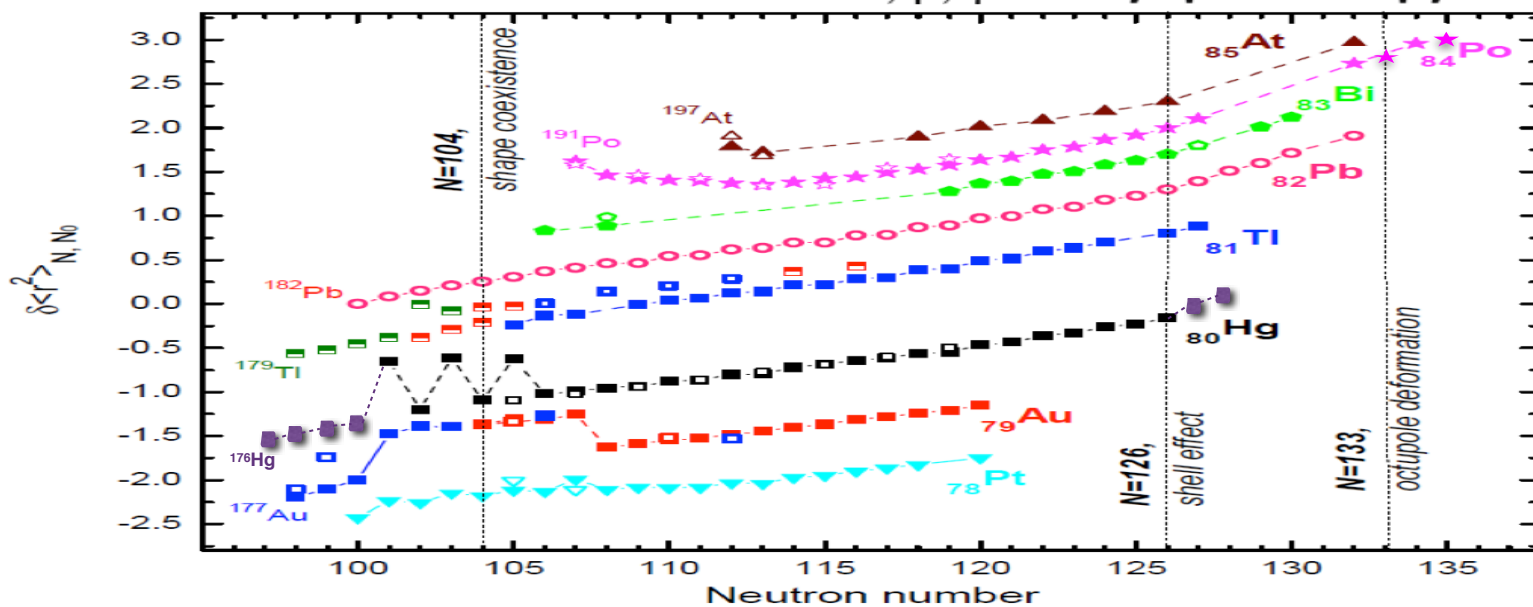
- pure plasma
- pure laser
- mixed plasma / laser

Everybody's favourite ion source

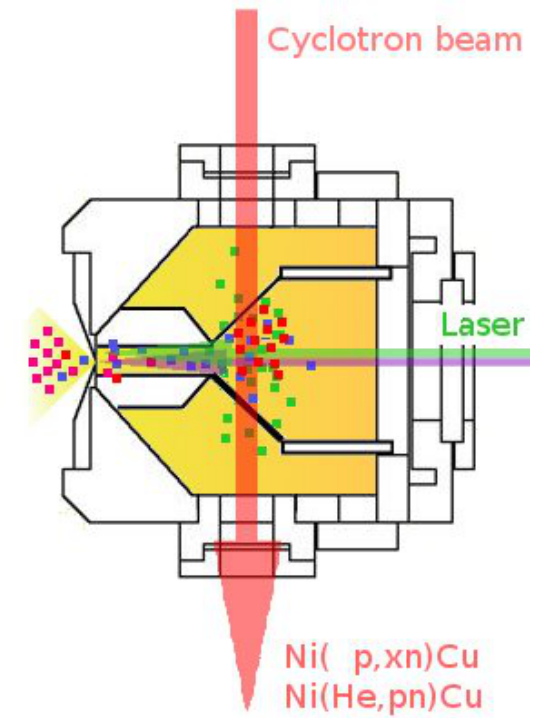
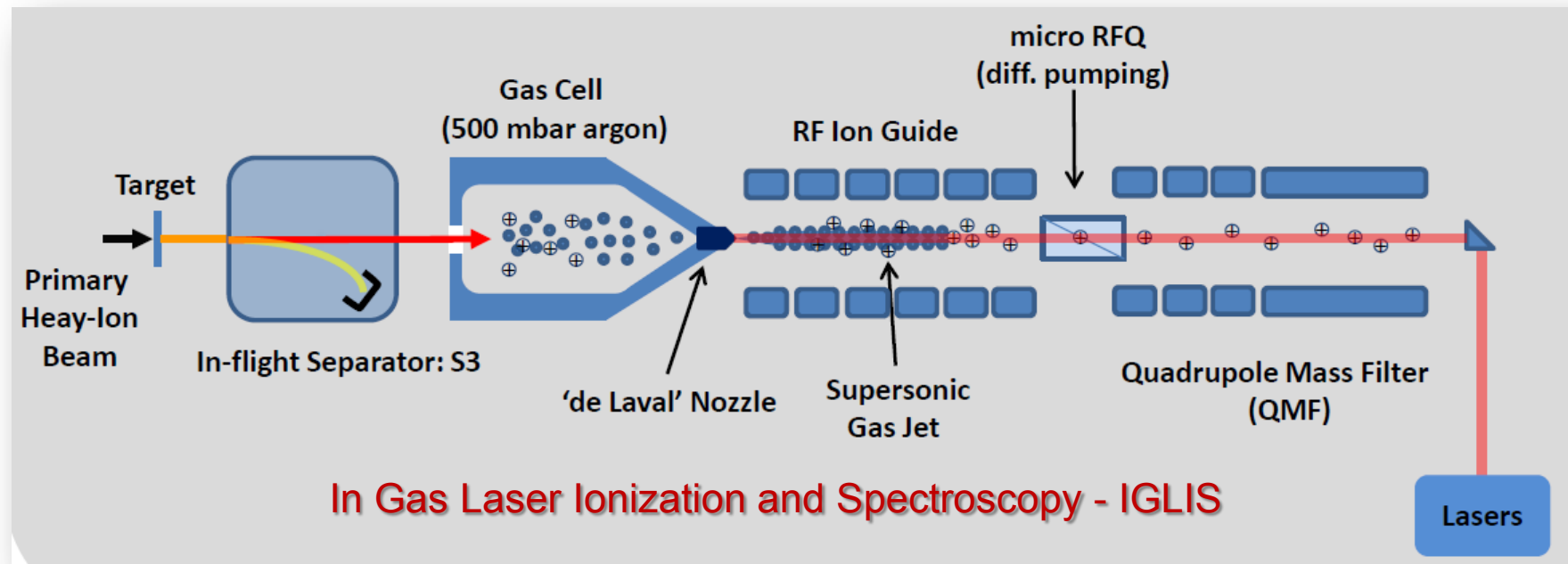
In-source spectroscopy



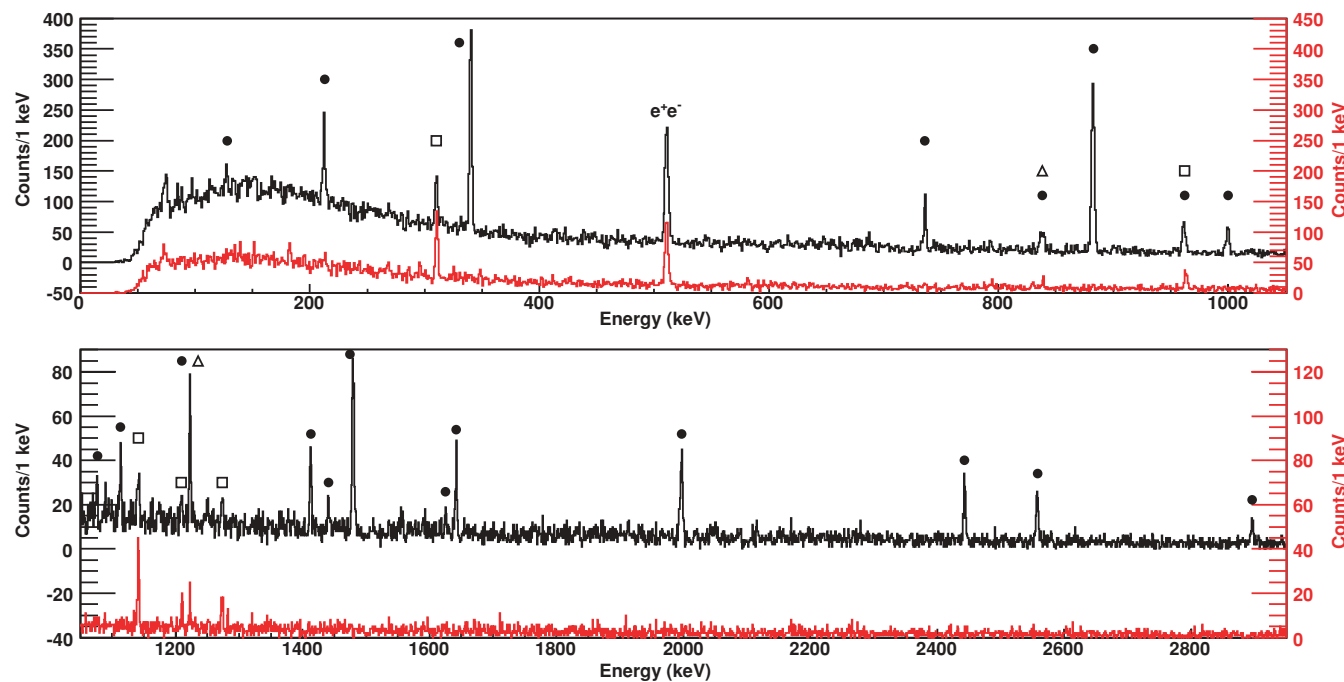
in laser spectroscopy



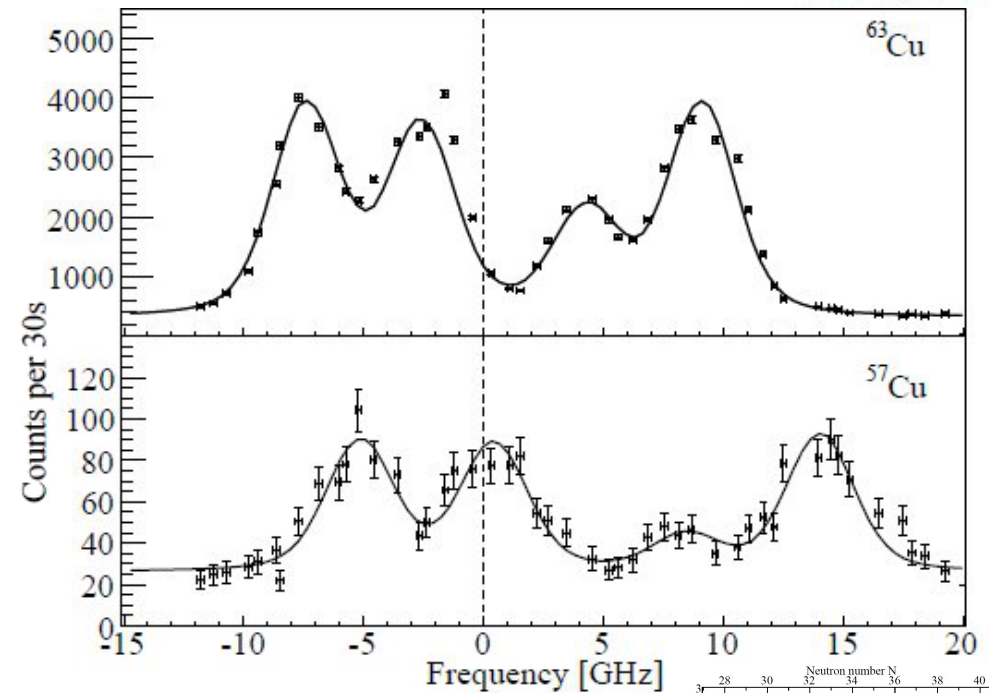
IGLIS



Fe vs Co

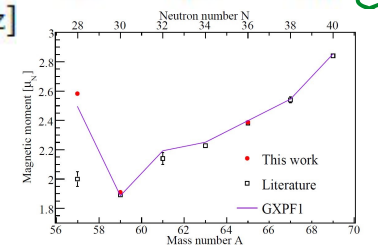


in β decay

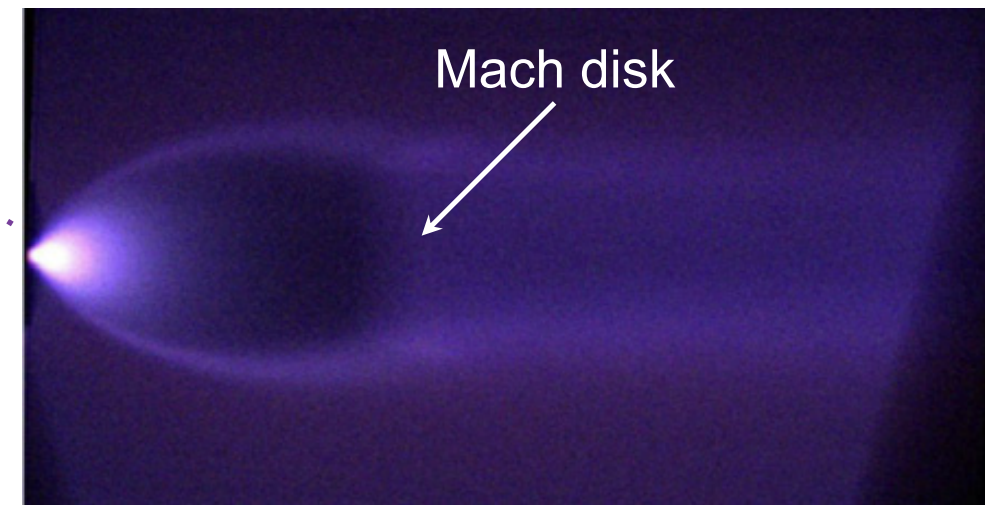
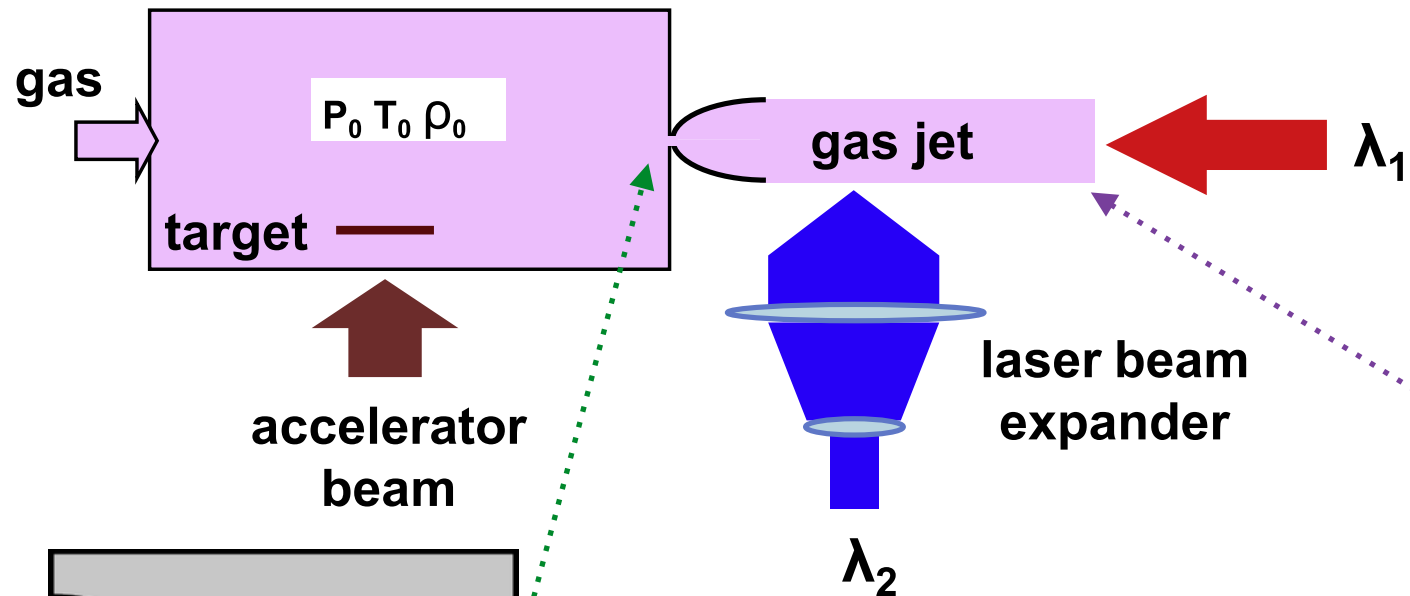


in laser spectroscopy

In-Gas Laser Ionisation & Spectroscopy

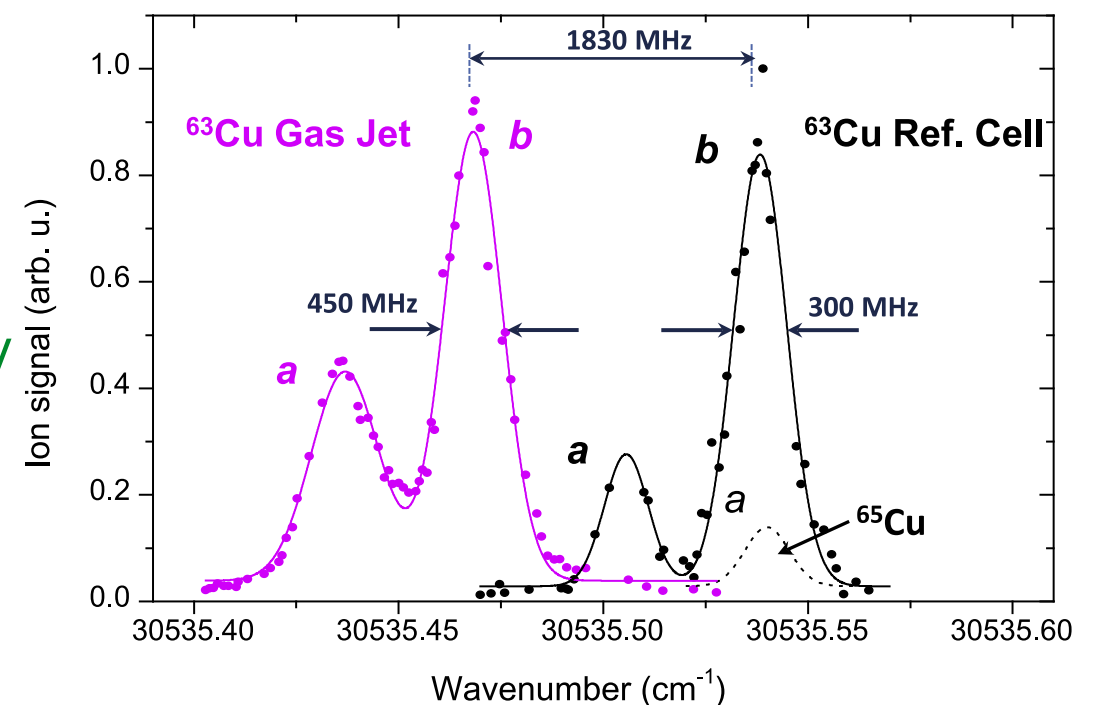
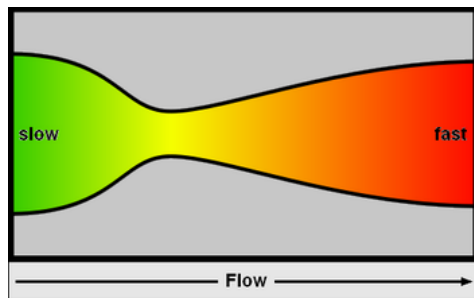


IJLIS



- ▶ Shaping a parallel gas jet for minimal divergence
- ▶ High Mach number for low temperature
- ▶ Cross beam geometry to probe only the jet

DeLaval Nozzle



In-Jet Laser Ion Source

Lecture 1

- The atom is sensitive to the properties of the nucleus
 - ▶ spin, electromagnetic moments, anomalies, distributions

- The laser ion sources provide selective enhancement
 - ▶ clean beams for experiments
 - ▶ isomeric beams for detailed studies
 - ▶ laser spectroscopy for dipole moments & charge radii



Ze END