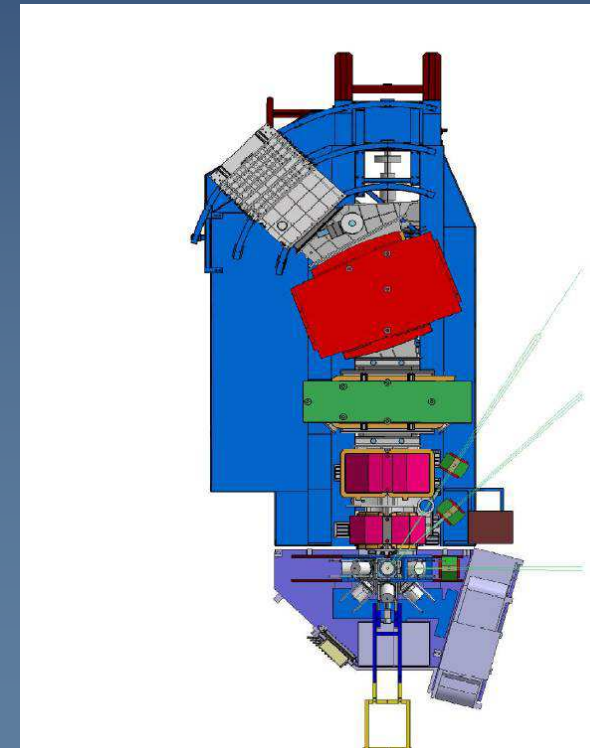
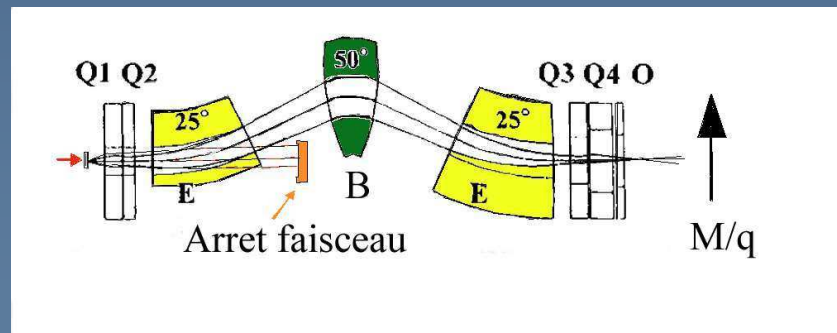
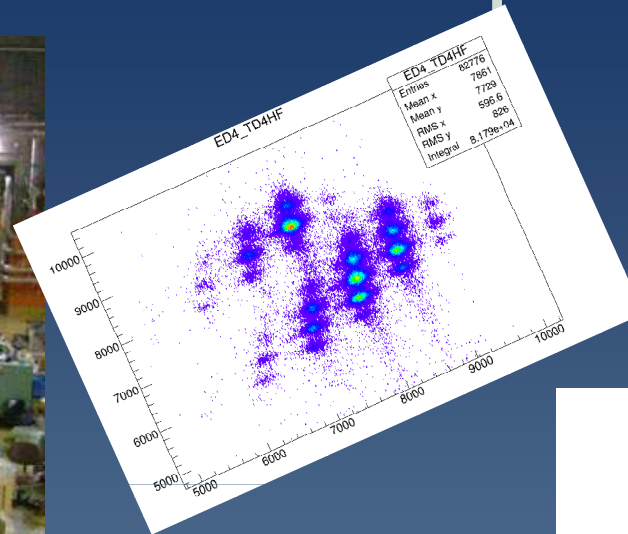


Electromagnetic Spectrometers & Separators



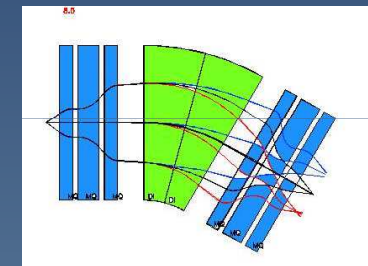
Spectrometers & separators

Properties in nuclear physics

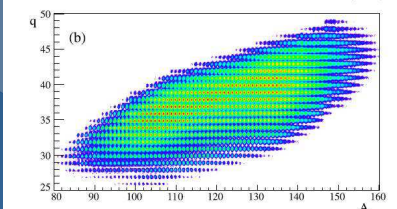
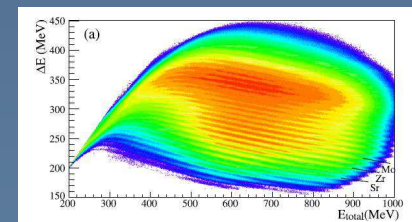
- 1) Why a spectrometer ? Part 1
- 2) Designing a spectrometer (1st approach)
- 3) Beam optics (Basics)
- 4) Spectrometer's **properties**

Part 2

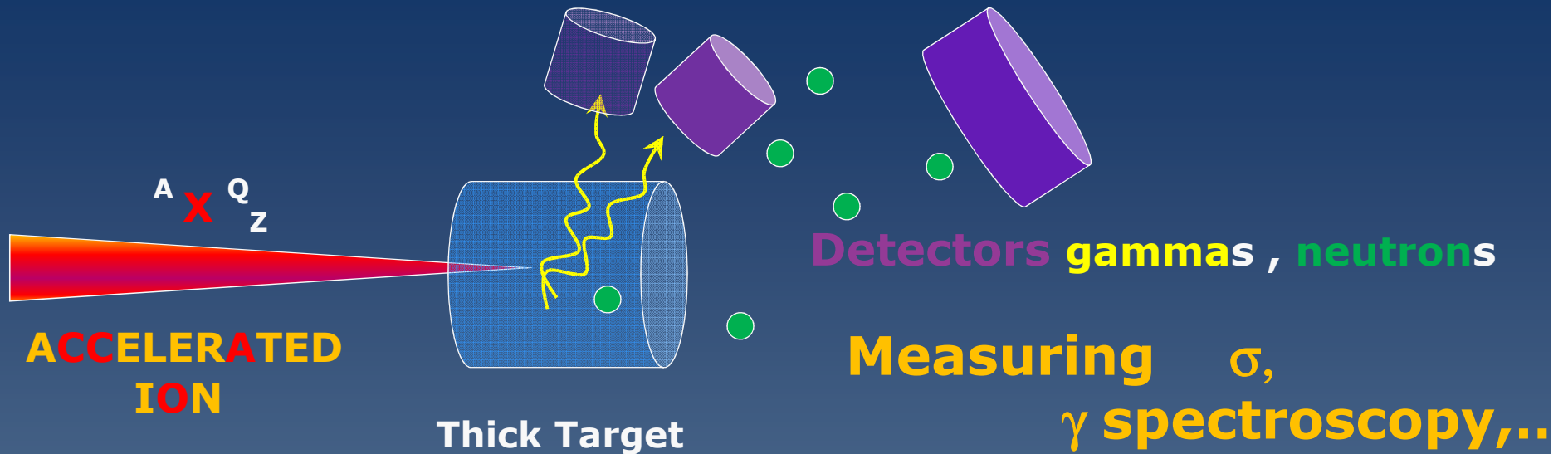
- 5) Fragments separators (100MeV/A-500 MeV/A)
- 6) Recoil Spectrometers (1-10 MeV/A)
- 7) Tuning And Diagnostics



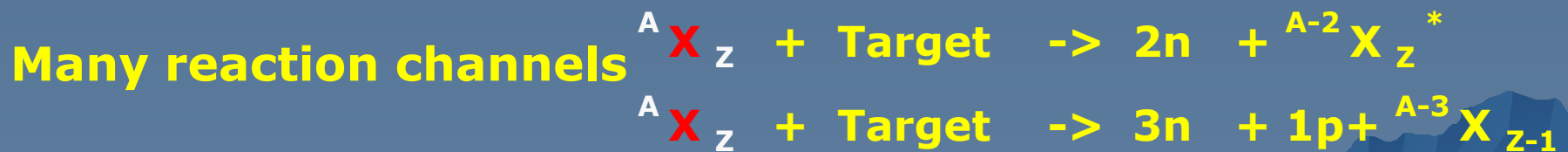
$$\begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



An experiment in nuclear physics

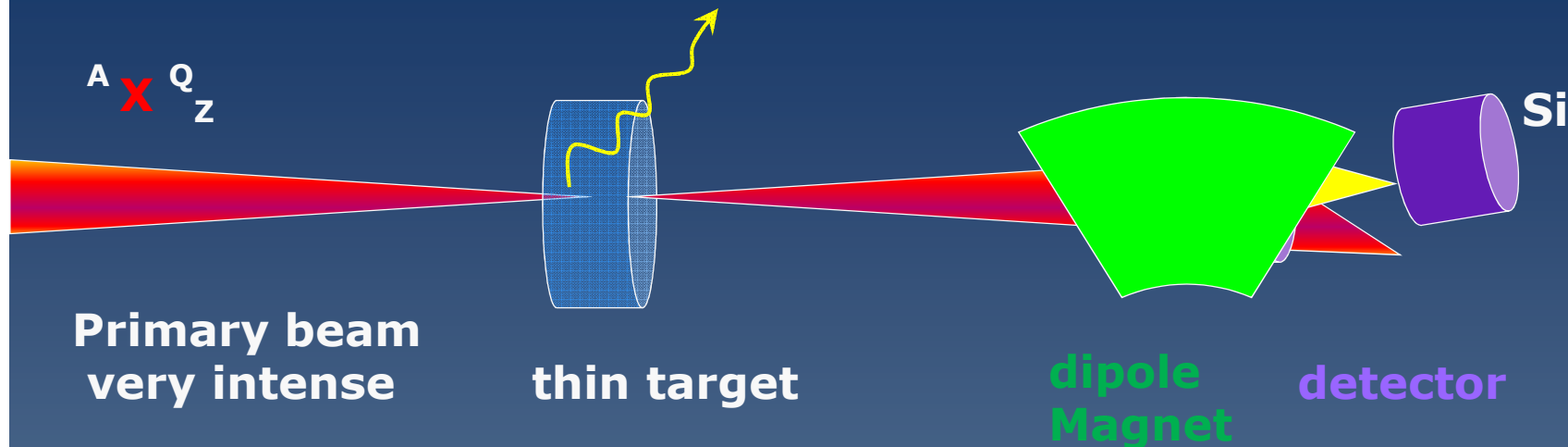


Reaction of interest, **but**



Reaction products not identified, ion energy not measured

An **other** experiment in nuclear physics



Electromagnetic spectrometer

- Eliminate **primary beam** ($\sim 10^{11-13}$ particles per second)
- Help to identify **the reaction products**
- Measure **Energy** with very good resolution
- Select **very rare events** (selectivity)

Is a **magnetic** spectrometer really needed ?

? **Complex question : YES and NO**

What observables do you need ?

with what resolution ?

(ion energy ,angle, A, Z, photon ,neutrons)

→ **with which detectors (position, energy)**

**Do you need to eliminate the Primary beam
from your detectors ?**

→ **primary beam separation**

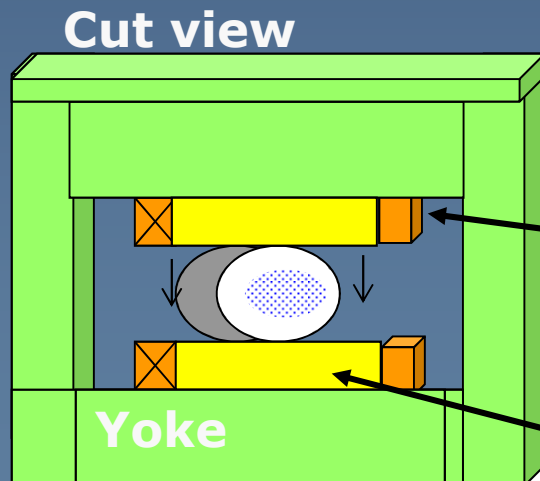
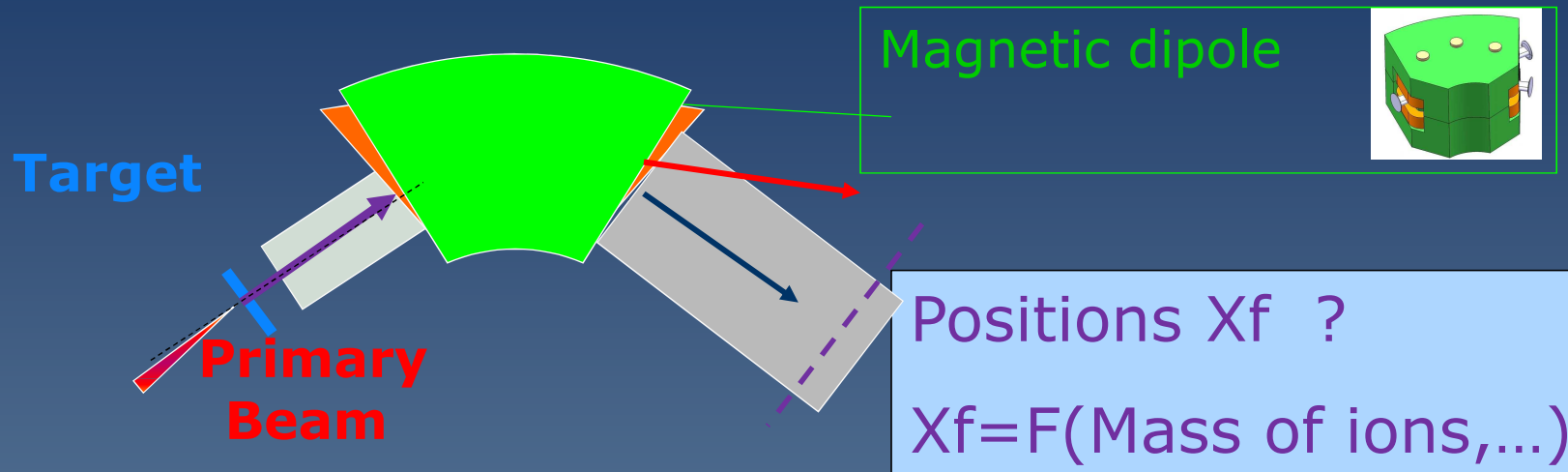
Without a magnetic spectrometer : limitation? (selectivity)

With a magnetic spectrometer : limitation ? (efficiency)

**Other possible limitations (primary beam intensity,
ion identification, detector resolution)**

Let's design a simple **Magnetic** spectrometer

1) dispersion of the particles as function of M, v, \dots



MAGNETIC DIPOLE : $B_y = \text{Constant}$
Coils (a current i induces and magnetic induction in the pole)

Yoke (guide the field lines to the pole)

2 flat poles : $B_y = \text{Constant}$

Equations for an ion *in a magnetic field* :

$$\vec{F} = q (\vec{v} \times \vec{B})$$

$$\frac{d}{dt} [\gamma m \vec{v}] = \vec{F}$$

$$d\vec{v}/dt = v^2 / R \mathbf{e}_r$$

Relativistic
factor :

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

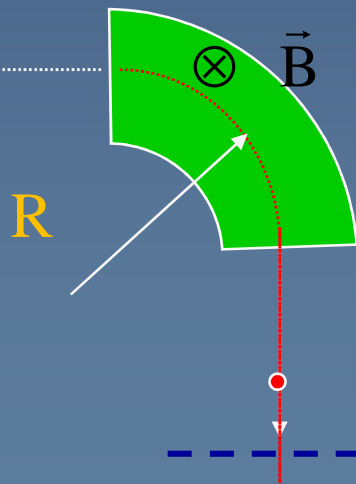
If $B = \text{const}$, for an ion (m, q, v)

The Trajectory is a circle with a radius R

$$R = \gamma \frac{mv}{qB}$$

We define the *magnetic rigidity* $B\rho = [\text{Tesla.m}]$

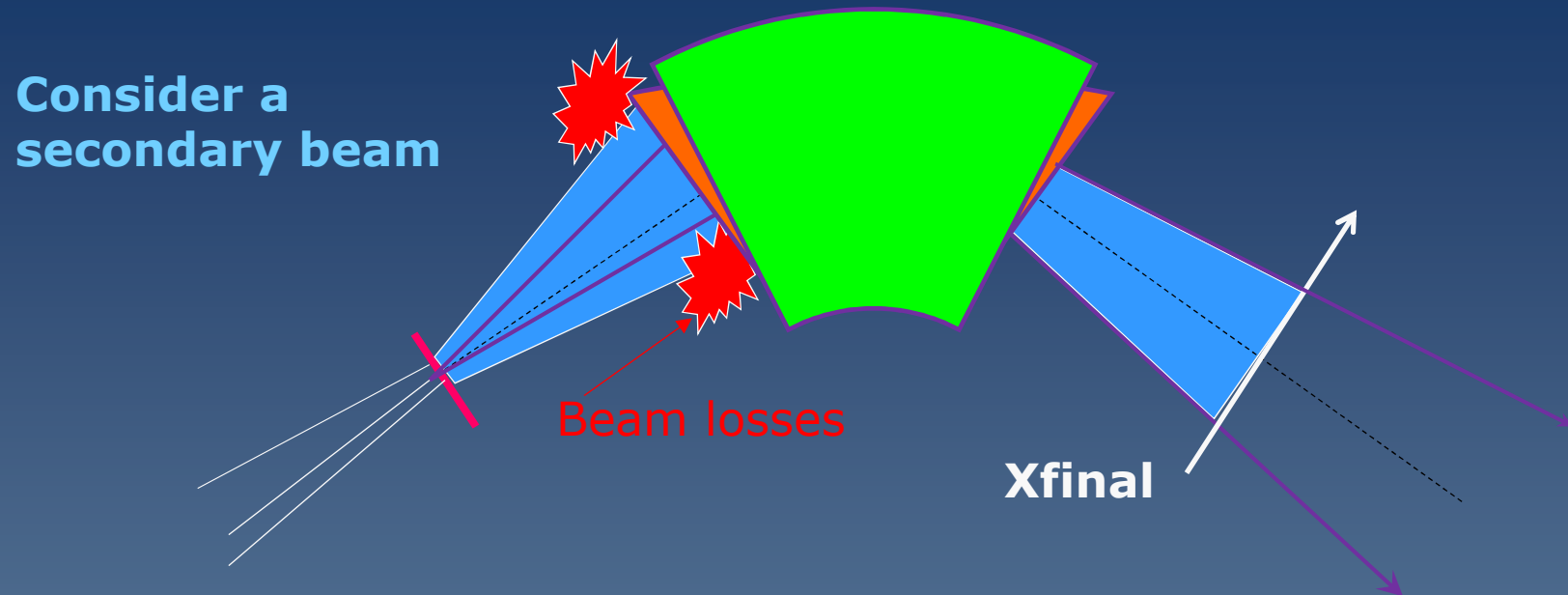
$$B\rho \stackrel{\text{def}}{=} \gamma \frac{mv}{q}$$



How to tune the dipole field $B = [\text{Tesla}]$?

$$B = \frac{B\rho_{\text{ref}} [T.m]}{R [m]}$$

2 problems with 1 dipole magnet : Acceptance & identification



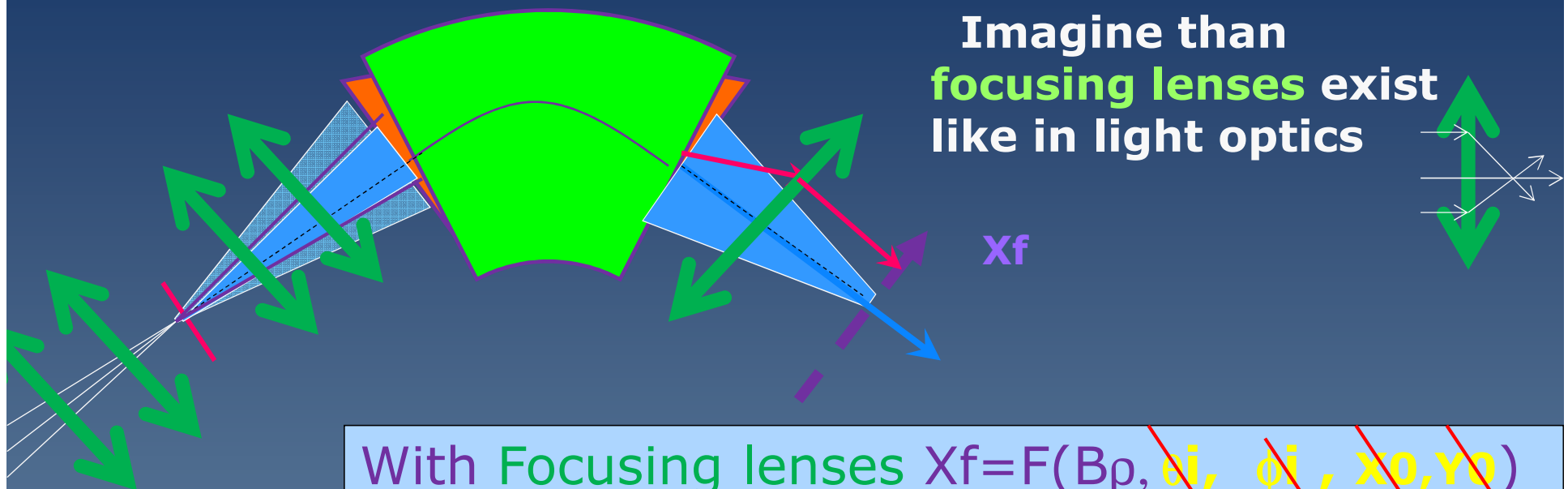
- 1: Many particles are lost in the magnet (**very bad**)
- 2: Trajectories are complex (**bad**)

$$X_{\text{final}} = f(B_{\rho}, \theta_i, \phi_i, X_0, Y_0)$$

- Final position x_f depend on the
 - B_{ρ} (good for identification or separation)
 - position & Angle after the reaction (**bad**)

Beam divergence after target

2 problems solved with **focusing lenses**



With Focusing lenses $X_f = F(B_p, \theta_i, \phi_i, x_0, y_0)$

less unknowns ! **Less beam losses!!**

At one location s (the detector location, called **focal plan**)

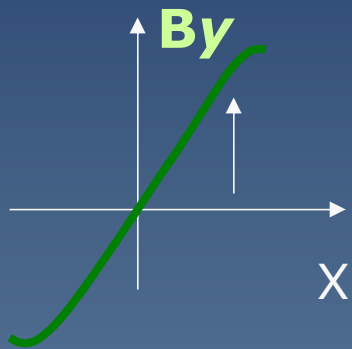
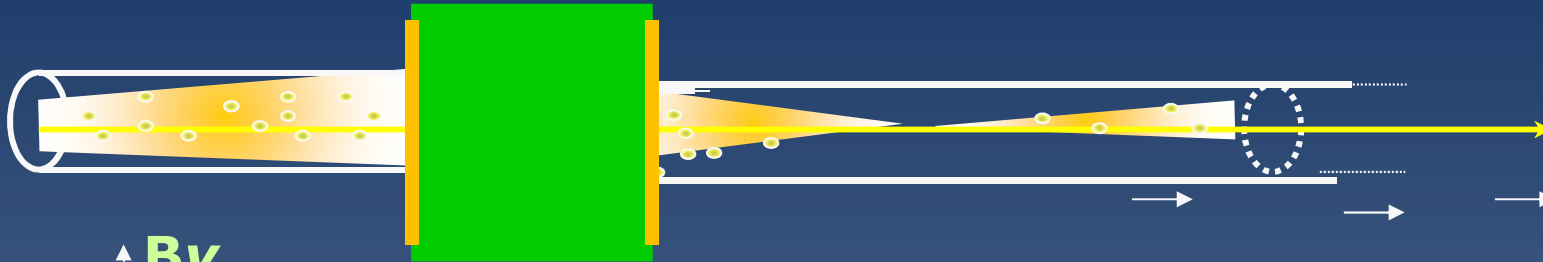
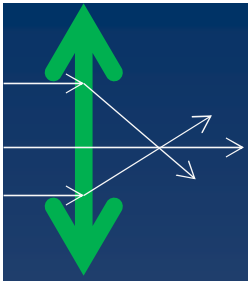
The trajectoires are independant of the angles θ_i, ϕ_i

And the initial position is $x_0=0, y_0=0$

$$X_f = F(B_p, \theta_i, \phi_i, x_0, y_0)$$

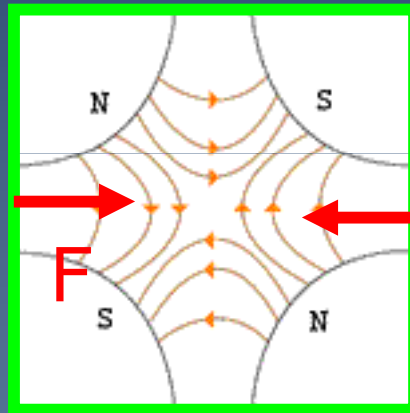
How to construct a *Focusing lens* for ions :
Magnet with 4 poles (+,-,+,-)

$$F = q (v \times B)$$



$$G = dBy/dx$$

L

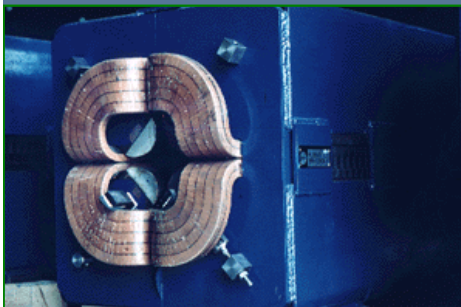


4 coils

+4 hyperbolic poles

$$By = G \cdot X$$

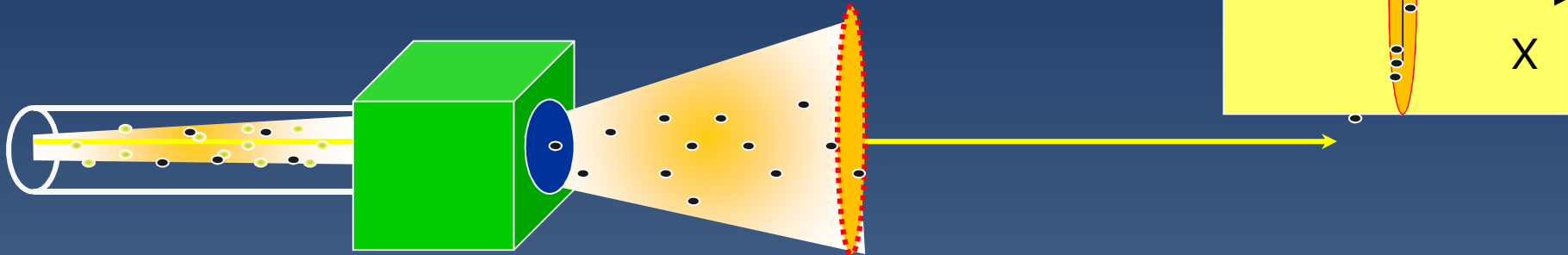
G is called GRADIENT
[Tesla/m]



The quadrupole magnet is **focusing**
in **HORIZONTAL PLAN**

Nota: In the center, the force is zero

*A quadrupole magnet
Focusing lens in horizontal
But defocusing in vertical*



The beam becomes narrow in X and large in Y

$$B_x = G \cdot Y$$

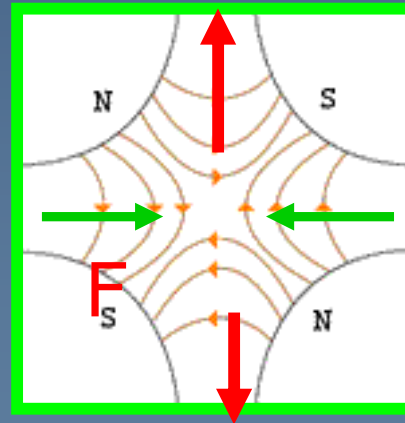
$$B_y = G \cdot X$$

$$B_s = 0$$

Focusing in X ($G > 0$)

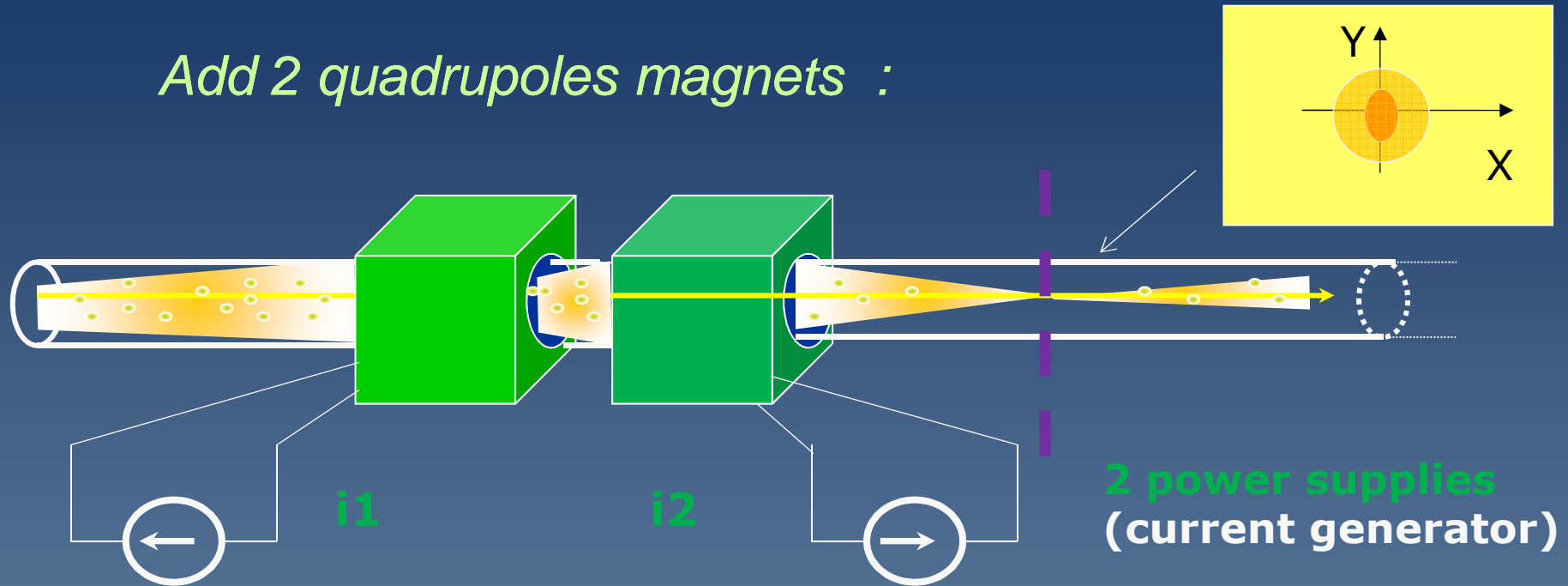
Defocusing in Y ($G > 0$)

Since the Force is defocusing in vertical



How to construct a Focusing lens System In horizontal AND vertical plan

Add 2 quadrupoles magnets :



If you tune i_1 and i_2 with opposite polarities , the beam can be focused in X and Y

Beam optics (basics)



◆ Focalisation with quadrupoles

DONE



▼ Dispersion with dipole

DONE

Magnetic rigidity : $B\rho = \gamma Mv/Q = P/Q$

DONE

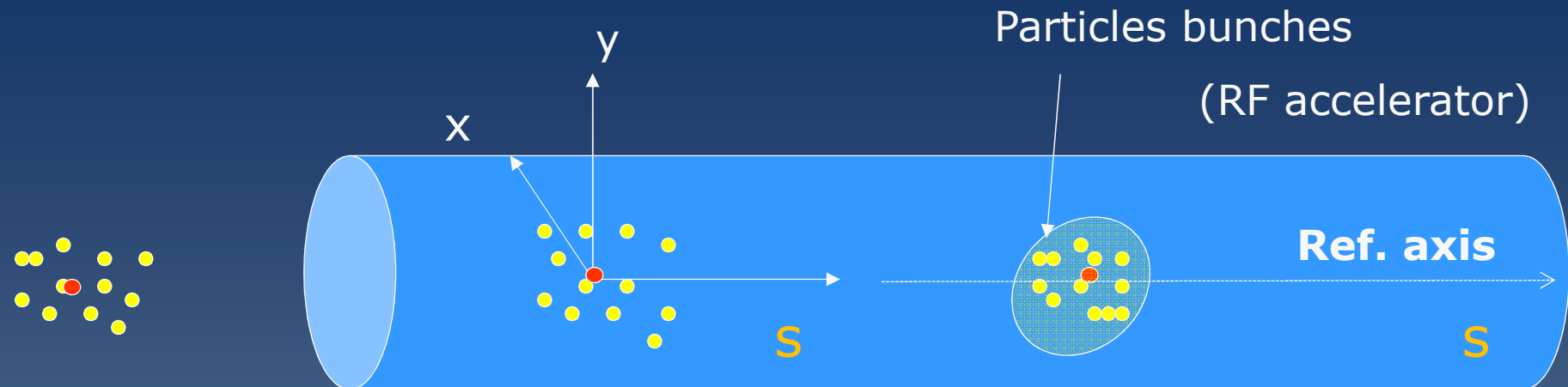
- Particles coordinates
- Equations in field B & E
- 1st order approximation : Optical Matrices



- Resolution
- Angular Acceptance
- $B\rho$ Acceptance



Beam Optics convention : Particle coordinates



particle coordinates ? (energy, velocity, angle, $B\rho$, ??)

DEFINE A REFERENCE PARTICLE ($x_0, y_0, B\rho_0, t_0$)

At a given S , a particle is described with **6 coordinates** :

2 positions ($X-X_0$), ($Y-Y_0$)

+ 2 angles θ, ϕ

+ rigidity $B\rho = B\rho_0 (1+\delta)$

+ ($t - t_0$) time advance

Optical convention :

Angle in Horizontal plan noted as

$$X' = dx/ds = \text{Tan } \theta$$

Angle in Vertical plan

$$Y' = dy/ds = \text{Tan } \phi$$

Time coordinate expressed in meter

$$L = v_0 (t - t_0)$$

Beam optics notation

The reference particle : $B\rho_0 = P_0/Q_0 = B_{dipole} \times R_{dipole}$

it is traveling in the **Center of the beam lines**

So $X_0=0$, $Y_0=0$

« angles » : $X'_0=0$, $Y'_0=0$

At the **location s_0** ,
a particle
 is represented
 by a vector $Z(s_0)$

$Z = (x, x', y, y', l, \delta)$
6Dim

$$\vec{Z} = \begin{pmatrix} z1 \\ z2 \\ z3 \\ z4 \\ z5 \\ z6 \end{pmatrix} = \begin{pmatrix} x \\ x' = \frac{dx}{ds} \\ y \\ y' = \frac{dy}{ds} \\ l = v_0(T - T_0) \\ \delta = \frac{B\rho - B\rho_0}{B\rho_0} \end{pmatrix} = \begin{pmatrix} \text{horizontal displacement} \\ \text{horizontal "angle" } \\ \text{vertical displacement} \\ \text{vertical angle} \\ \text{longitudinal difference} \\ \text{"momentum}(B\rho)\text{" deviation} \end{pmatrix}$$

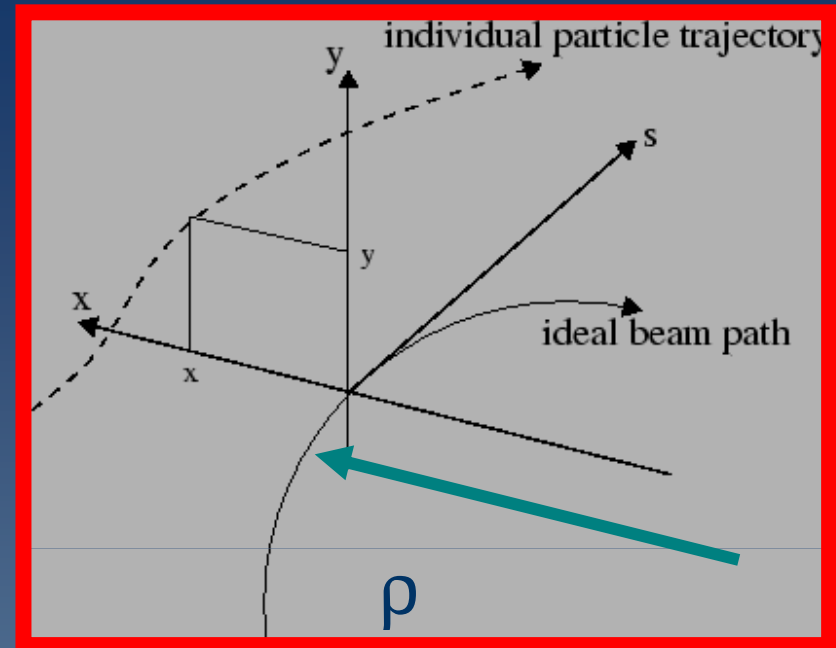
Trajectory equations for 1 particle

How to compute $x(s), y(s)$?

We use a

curvilinear Reference Frame

which follow the reference particle



$$\frac{d}{dt} [m\gamma \mathbf{v}] = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{d}{dt} = \dot{s} \frac{d}{ds}$$

Coordinate change $t \Rightarrow s$

$x(t), y(t) \Rightarrow x(s), y(s)$

We want to compute x, y at a detector location $s=s_0$

$$\frac{d}{ds} [m\gamma \mathbf{v}] = \dots$$

Trajectories : exact equations

$$\frac{d}{ds} \left[m\gamma \dot{x} \right] = m\gamma \dot{s} \left(1 + \frac{x}{\rho} \right) + q(t' E_x + y' B_s - \dot{s} \left(1 + \frac{x}{\rho} \right) \cdot B_y)$$

$$\frac{d}{ds} \left[m\gamma \dot{y} \right] = q(t' E_y + \left(1 + \frac{x}{\rho} \right) \cdot B_x - x' \cdot B_s)$$

$$\frac{d}{ds} \left[m\gamma \dot{s} \left(1 + \frac{x}{\rho} \right) \right] = -\frac{m\gamma \dot{x}}{\rho} + q(t' E_s + x' \cdot B_y - y' \cdot B_x)$$



Trajectory simulation ($x(s)$, $y(s)$)

- 1) knowing $B(x,y,s)$ AND $E(x,y,s,t)$ [field map 3D]
- 2) Integrate the equations for **ALL the particles** (computer+ Numerical method: Runge-kutta)

Generally we can do simpler

Matrix approach (1st order approximation)



Beam optics with Matrices



$$\begin{aligned} \mathbf{Z}_2 &= f_{1 \rightarrow 2} (\mathbf{Z}_1, B, E, l, \dots) \\ &= R_{1 \rightarrow 2} \cdot \mathbf{Z}_1 + O(\mathbf{Z}_1^2) + \dots \\ &\approx R_{1 \rightarrow 2} \cdot \mathbf{Z}_1 \end{aligned}$$

Exact Dynamic (non linear)

Taylor expansion

(X and Y Are small..)

Linear dynamics

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_2 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1$$

$$\begin{aligned} l &= v_0(t - t_0) \\ \delta &= \frac{B\rho - B\rho_0}{B\rho_0} \end{aligned}$$

The transport Matrix R : allow the computation of a coordinate of a particle at the end of a spectrometer

$$\begin{array}{c} \longrightarrow \\ Z_{in} = (x, x', y, y', l, \delta)_0 \end{array} \quad \text{at the entrance}$$

$$\begin{array}{c} \longrightarrow \\ Z_{out} = (x, x', y, y', l, \delta)_1 \end{array} \quad \text{at the exit}$$

$$\begin{array}{c} \longrightarrow \quad \longrightarrow \\ Z_{out} = R \cdot Z_{in} \end{array}$$

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1 = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{31} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & R_{55} & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_0$$

$$l = v_0(t - t_0)$$

$$\delta = \frac{p - p_0}{p_0}$$

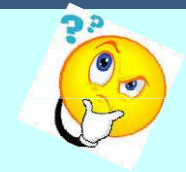
Interpretation of R

$$R_{ij} = \left(\frac{\partial Z_i \text{ out}}{\partial Z_j \text{ in}} \right)$$

ex:

$$R_{11} = \left(\frac{\partial Z_1}{\partial Z_1} \right) = \left(\frac{\partial x \text{ out}}{\partial x \text{ in}} \right) \quad R_{12} = \left(\frac{\partial Z_1}{\partial Z_2} \right) = \left(\frac{\partial x \text{ out}}{\partial x' \text{ in}} \right)$$

$$R_{16} = \left(\frac{\partial Z_1}{\partial Z_6} \right) = \left(\frac{\partial x \text{ out}}{\partial \delta \text{ in}} \right)$$



The transport Matrix $R = R_{ij}$ is related to

- spectrometer geometry
- **tuning** of the quadrupoles

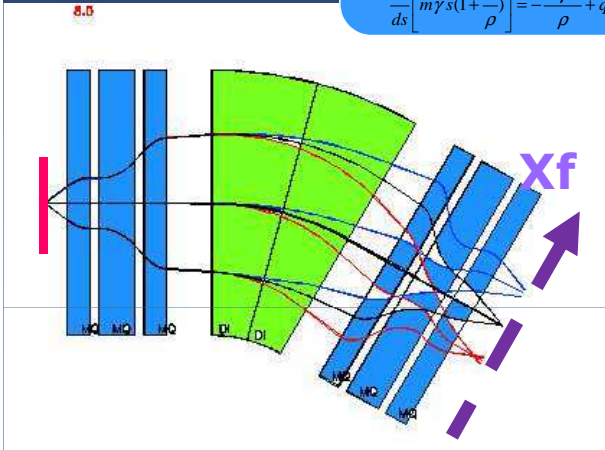
SPECTROMETER TRANSPORT MATRIX R

allow the simulation of 1 trajectory (easily)

$$\frac{d}{ds} \left[m\gamma \dot{x} \right] = m\gamma \dot{s} \left(1 + \frac{x}{\rho} \right) + q(t' E_x + y' B_s - \dot{s} \left(1 + \frac{x}{\rho} \right) \cdot B_y)$$

$$\frac{d}{ds} \left[m\gamma \dot{y} \right] = q(t' E_y + \left(1 + \frac{x}{\rho} \right) \cdot B_x - x' B_s)$$

$$\frac{d}{ds} \left[m\gamma \dot{s} \left(1 + \frac{x}{\rho} \right) \right] = -\frac{m\gamma \dot{x}}{\rho} + q(t' E_s + x' B_y - y' B_x)$$



$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_{FINAL} = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_{TARGET}$$



Typical spectrometer Matrix is simple

$$\mathbf{X}_{Final} = R_{11} \mathbf{X}_{target} + R_{16} \delta$$

$$\approx R_{16} \delta$$

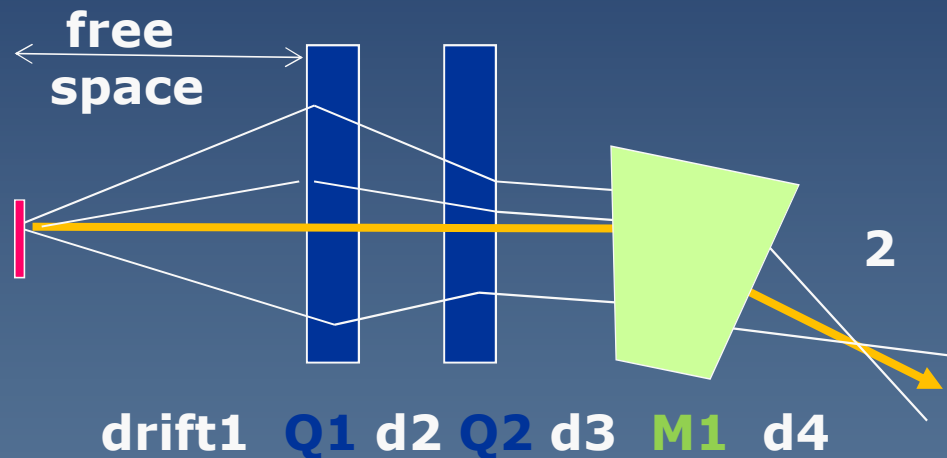
$$R_{16} = \left(\frac{\partial x_F}{\partial \delta_{Target}} \right)$$

$$\delta = (B\rho - B\rho_0) / B\rho_0$$

$$B\rho_0 = \mathbf{B}_{dipole} \cdot R_{dipole}$$

More on Transport Matrices: how to compute the Rmatrix for a spectrometer ?

The **total transport** matrix R is the **product** of the matrices representing each elements (drift ,quad, dipole)



Quad matrix

$$R_{M1} = \begin{bmatrix} \cos k_x L & \frac{\sin k_x L}{k_x} & 0 & 0 & 0 & M_{16} \\ -k_x \sin k_x L & \cos k_x L & 0 & 0 & 0 & M_{26} \\ 0 & 0 & \cos k_y L & \frac{\sin k_y L}{k_y} & 0 & 0 \\ 0 & 0 & -k_y \sin k_y L & \cos k_y L & 0 & 0 \\ M_{26} & M_{16} & 0 & 0 & 1 & M_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$k_x = GL / B\rho_0$$

$$R_{d1} = \begin{bmatrix} 1 & L1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L1/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Free space:
Drift Matrix

Matrix product

$$R = d4 \cdot R_{M1} \cdot R_{d3} \cdot R_{Q2} \cdot R_{d2} \cdot R_{Q1} \cdot R_{drift1}$$

The **beam size** : important for the design

- A **particle** has 1 trajectory : $Z = \vec{Z}(s)$

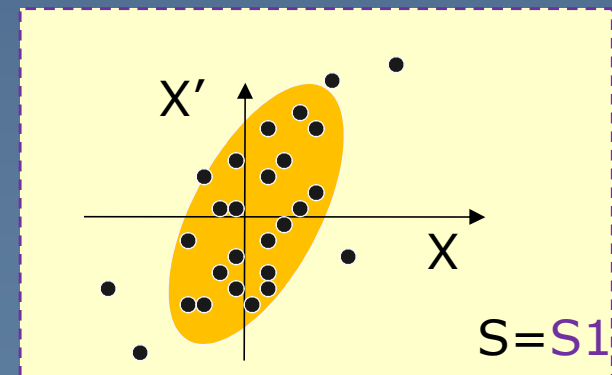
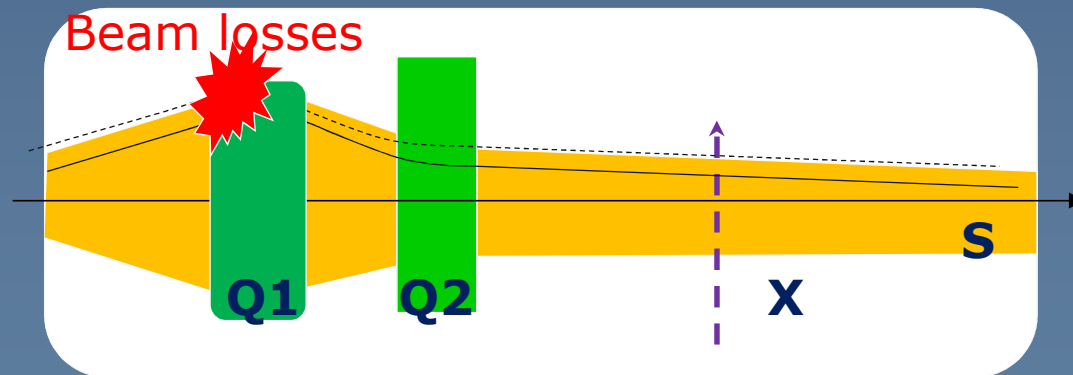
We are not interested by only 1 trajectory/particle

*A beam is an ellipsoid in 6D with a **given size***

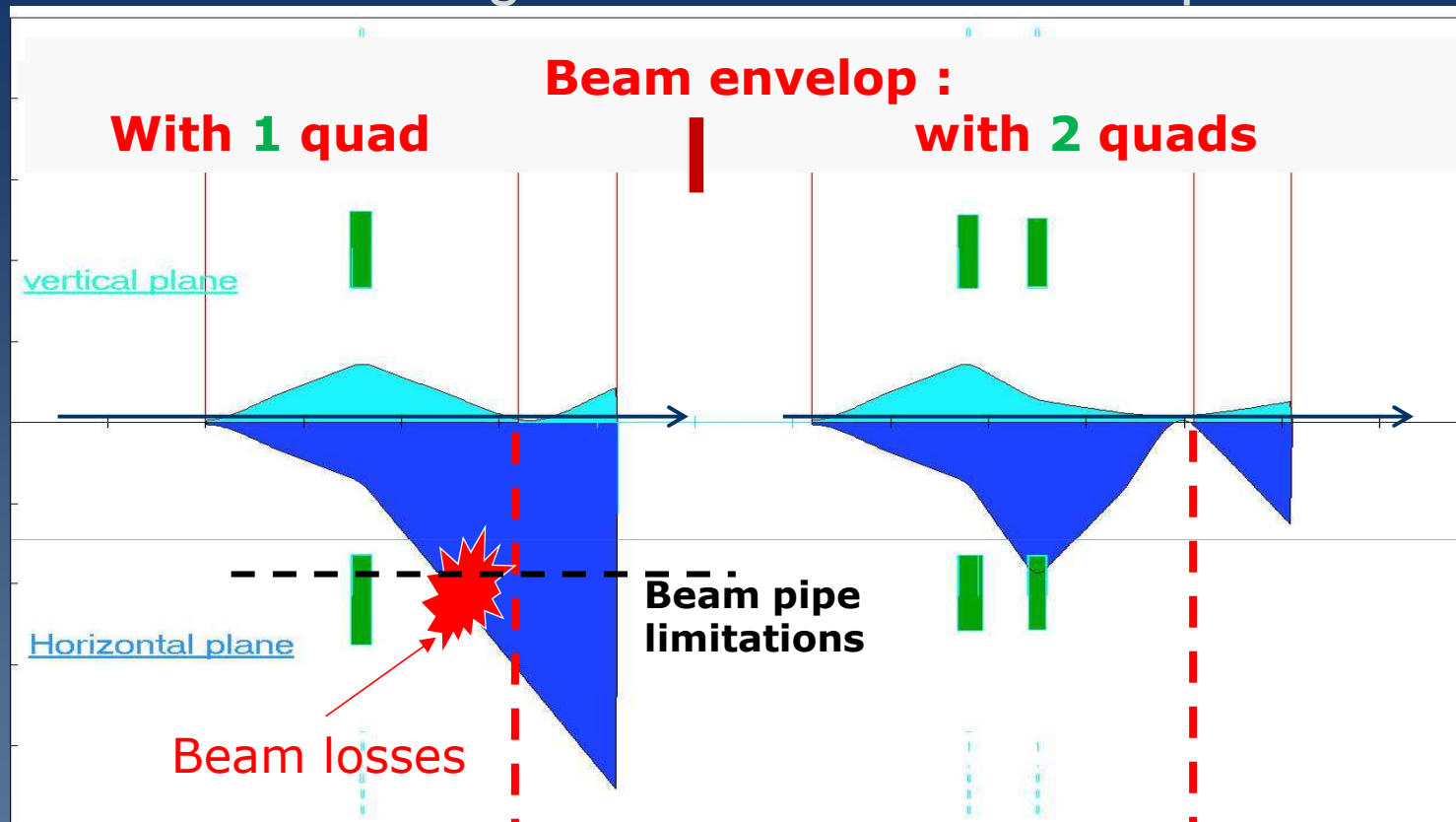
*The **beam size(width)** has to be simulated to avoid **beam losses***

σ_x (horizontal width) , σ_y (vertical width)

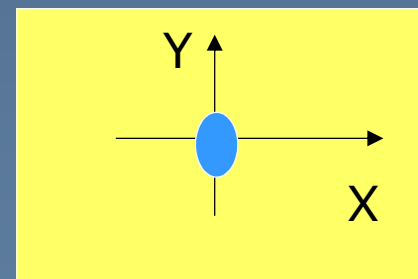
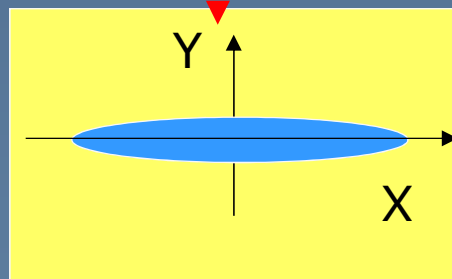
$$\sigma_x^2 = \frac{1}{N} \sum_{\alpha=1, \dots, N} x_\alpha^2$$



Focusing a beam in a simulation get a small size at some point S



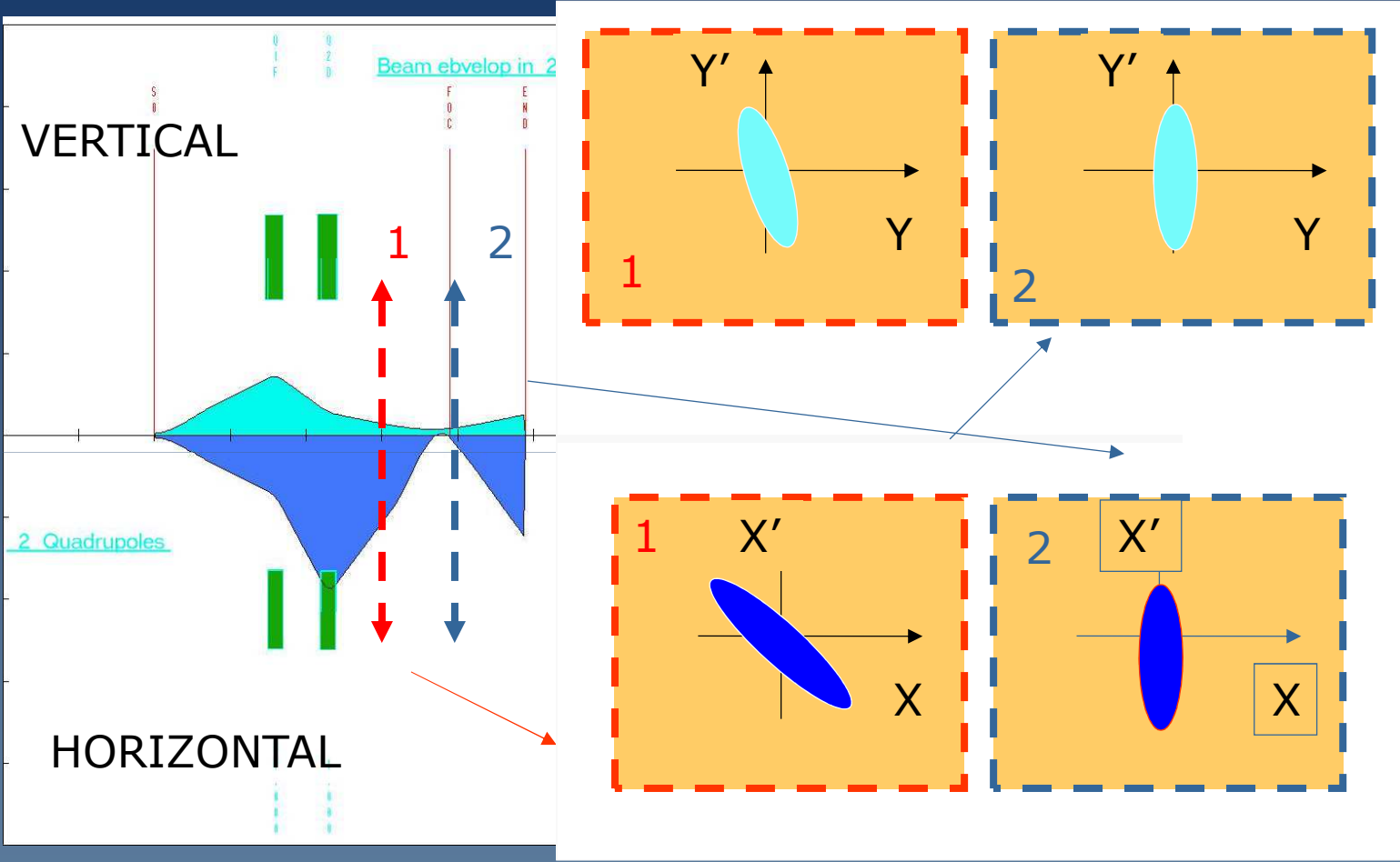
Adjust
Quad gradient
 $G_q = dB_y/dx$



Focusing in X and Y : at less 2 quads required

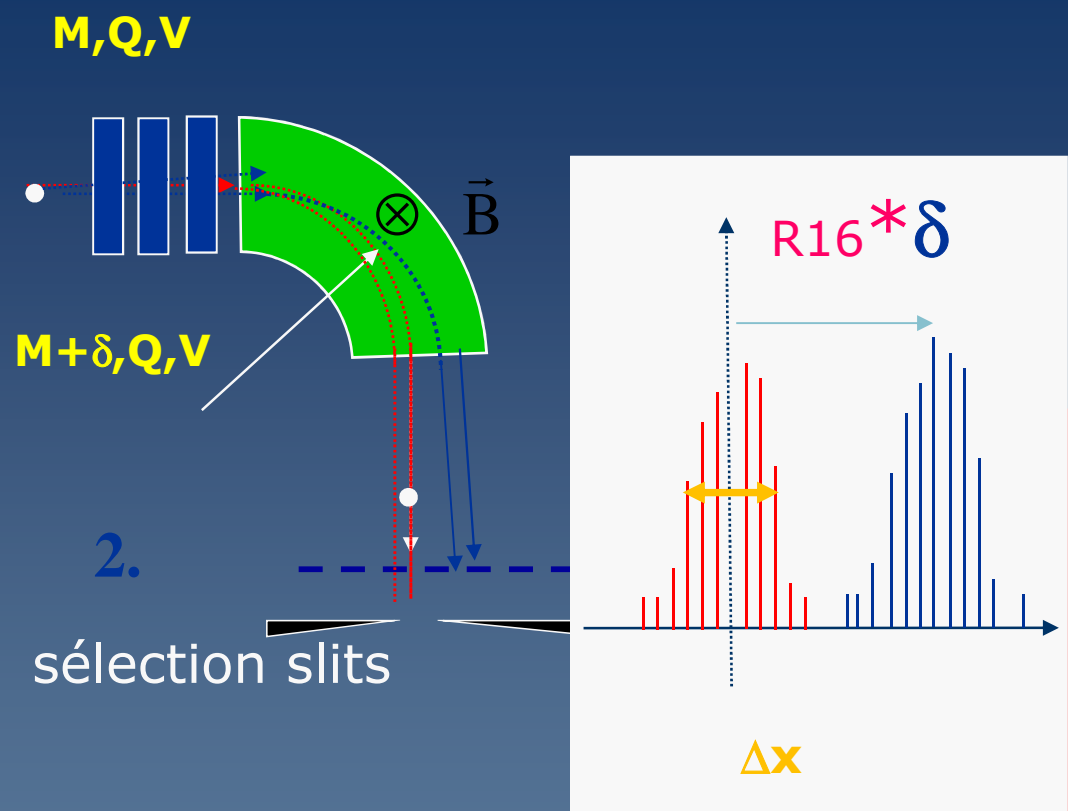
Angular distribution (x') in a beam line ?

The beam ellipse is rotating in $(x, x'=dx/ds)$



...The Area of the beam ellipse (x, x') is a constant in a beam line... but, Area is not constant in a target

Resolution of a separator



particles are separated

IF $R_{16} * \delta > 4 \sigma_x$

Resolution = $4 \sigma_x / R_{16}$
 = Minimal difference in $B\rho$
 for the identification
 or for separation

$R_{16} = \left(\frac{\partial Z_1}{\partial Z_6} \right) = \left(\frac{\partial x_{out}}{\partial \delta_{in}} \right)$

R=1/100 Resolution means :
 capacity for a spectrometer to
 distinguish two beams with
 1% $B\rho$ difference

The resolution (separation)

is **optimal** at the **focus point** (size is minimal)

The 2 beams with \neq rigidities

$$B\rho_{\text{ref}} = B\rho_0 = B \times R_{\text{dipole}}$$

$$B\rho = B\rho_0(1-\delta)$$

The 2 beams are separated

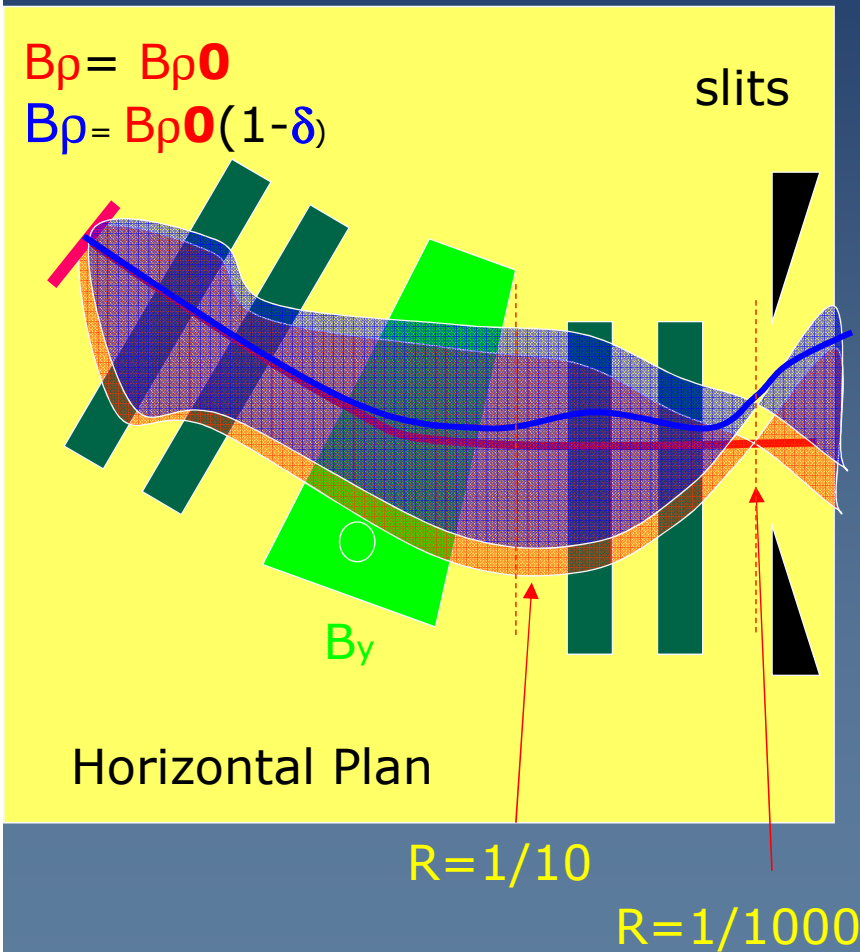
« at the focal plan »

But not everywhere !!

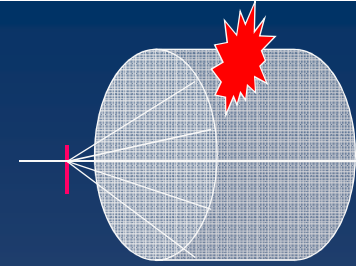
Resolution ($R = \sigma_x / R16$) is **optimal**

When σ_x is **small**

and $R16$ (dispersion) is **large**

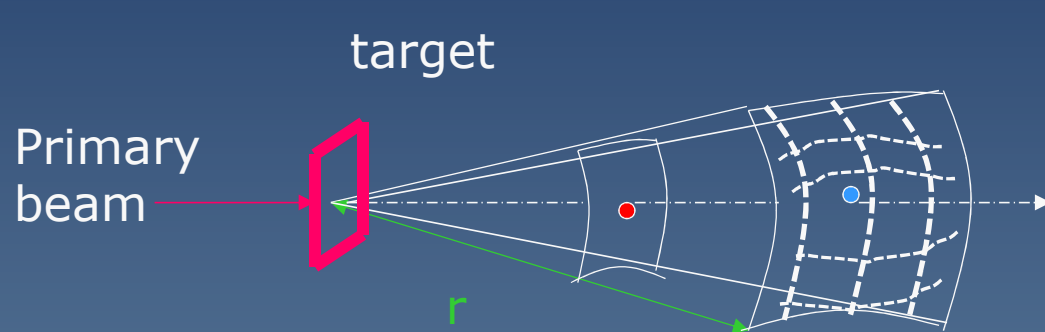


Angular acceptance



The **reaction products** exit from the target with an **Angular dispersion**

Vacuum chamber limitation induces **beam losses** = less transmission



The acceptance is measured in steradian.

$$d\Omega(\text{strd}) = \frac{dS}{r^2}$$

dS

Example: If particles inside **$\pm 50\text{mrd}$** (Horizontal & vertical) are transmitted

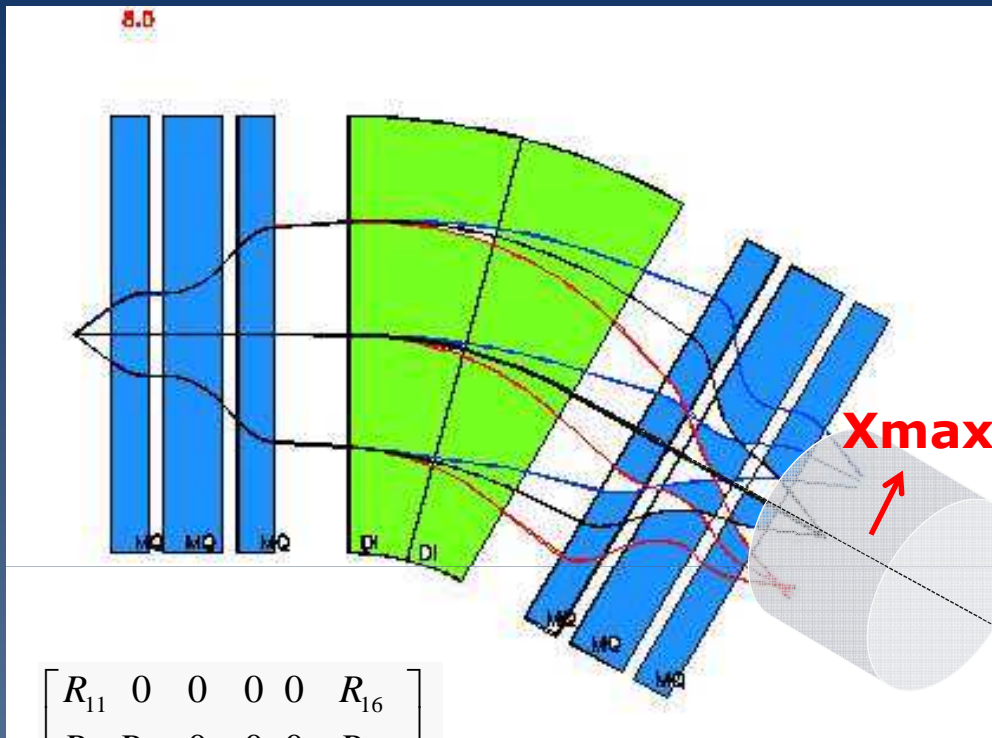
Acceptance is $d\Omega \approx 0.01\text{strd} = 10 \text{ mstrd}$

at $r=1\text{m}$

$$dS \# 0.1\text{m} * 0.1\text{m} = 0.01 \text{ m}^2$$



« B_ρ » Acceptance



The particles are dispersed by dipole magnets with $\delta = [B_{\rho} - B_{\rho 0}] / B_{\rho 0}$

$$X_{final} = R_{16} \delta$$

Beam pipe limit: X_{max}

$$\begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_{\rho} \text{ Acceptance} = \pm X_{max} / R_{16}$$

Example : If $R_{16} = 5 \text{ cm}/\%$ and $X_{max} = 10 \text{ cm}$

$$B_{\rho} \text{ Acceptance} = \pm 2 \%$$

How to simulate an experiment with a spectrometer

The screenshot displays the LISE++ software interface. On the left, a vertical panel lists experimental components: Projectile (78Kr³⁶⁺), Fragment (32S¹⁶⁺), Target (Be, 500 micron), Stripper, Dipole 1 (Brho, 2.2308 Tm), PPAC (Al, 1 micron), Wedge, Dipole 2 (Brho, 2.2305 Tm), and Material 2 (Si, 300 micron). The main area features a periodic table with a 'PROJECTILE FRAGMENT' diagram overlaid. A 'Physical calculator' window is open, showing input parameters for Element S (Z=16, Q=16) and output values for Energy (12.0044 MeV/u), Brho (1.0000 Tm), and Energy Loss (383.81 MeV). The calculator also displays 'Range and Energy Loss to Si' with values for Range (36.8937) and dRange (0.089295 mg/cm2).

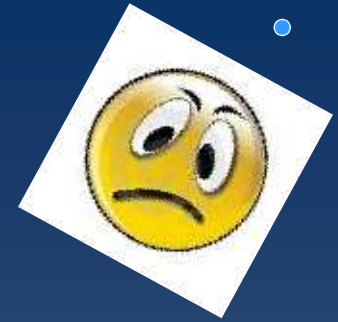
LISE++

code*

Tarasov et Al.

**To be
Downloaded**

HOMEWORK :



Exercise 1: Imagine a spectrometer with a dispersion $R_{16}=2 \text{ m } (=2\text{cm}/\%)$ and beam width $\sigma_x = 0.5 \text{ mm}$ on the focal plan detector,
What is the resolution R in B_ρ ?

Exercise 2 :

A spectrometer ($R_{16}=1.5 \text{ cm}/\%$) is tuned for $B_{\rho 0}=2.0 \text{ T.m}$
A particle arrives on the focal plane at $X_f=3\text{cm}$,
What is the particle rigidity?

Exercise 3 :

How to measure the dispersion (R_{16}) in a spectrometer ?

Part 1 :

- The need of focalisation (quad)
- Magnetic rigidity define the trajectory
- Dynamics can be approximated with a matrix R

$$B\rho \stackrel{\text{def}}{=} \gamma \frac{mv}{q}$$

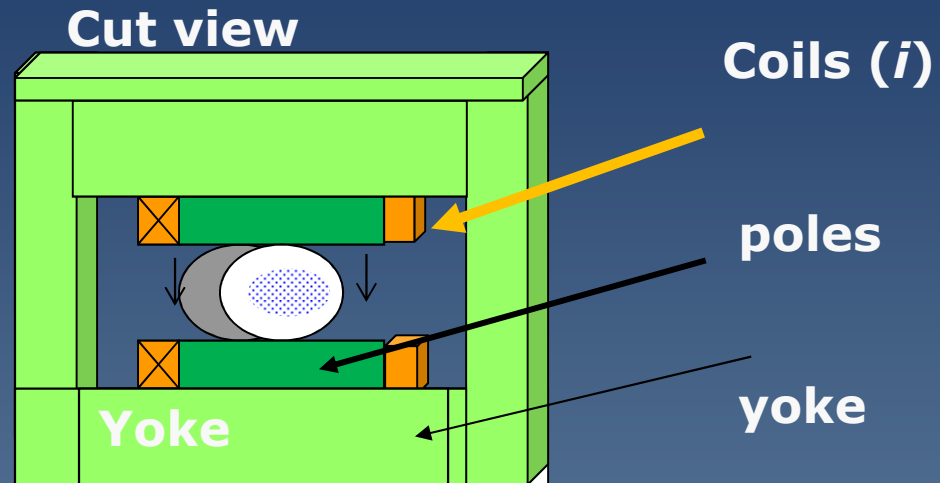
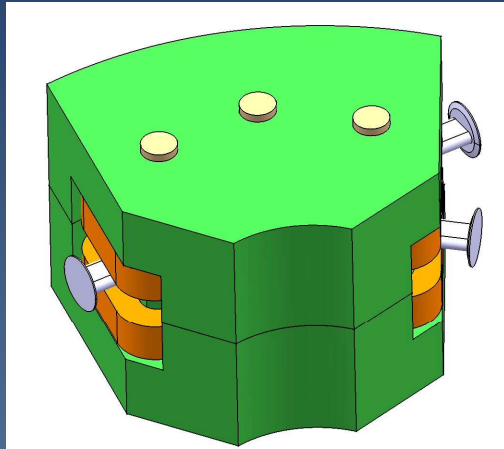
End part n°1

Part 2 : technical details and examples

- Resume of part 1
- Fragment separators ($E > 100$ MeV/A)
- Recoil Spectrometers ($E < 10$ MeV/A)
- Diagnostics and tuning

Part n°1 : Spectrometer components

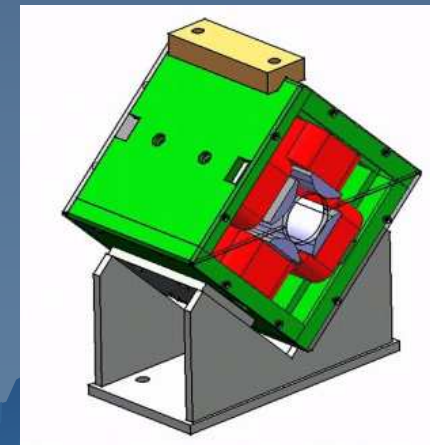
Magnetic Dipole : 2 poles : $B_y = B_0$



Magnetic quadrupole : 4 poles

$$B_y = G x$$

focusing is good
for **Angular acceptance** and **Resolution**



Beam optics coordinates

- ◆ At the location S , a particle is represented by a vector $\mathbf{Z}(s) = (x, x', y, y', l, \delta)$

$$\vec{Z} = \begin{pmatrix} z1 \\ z2 \\ z3 \\ z4 \\ z5 \\ z6 \end{pmatrix} = \begin{pmatrix} x \\ x' = \frac{dx}{ds} \\ y \\ y' = \frac{dy}{ds} \\ l = v_0(T - T_0) \\ \delta = \frac{B\rho - B\rho_0}{B\rho_0} \end{pmatrix} = \begin{pmatrix} \text{horizontal displacement} \\ \text{horizontal "angle"} \\ \text{vertical displacement} \\ \text{vertical angle} \\ \text{longitudinal difference} \\ \text{"momentun}(B\rho)" \text{ deviation} \end{pmatrix}$$

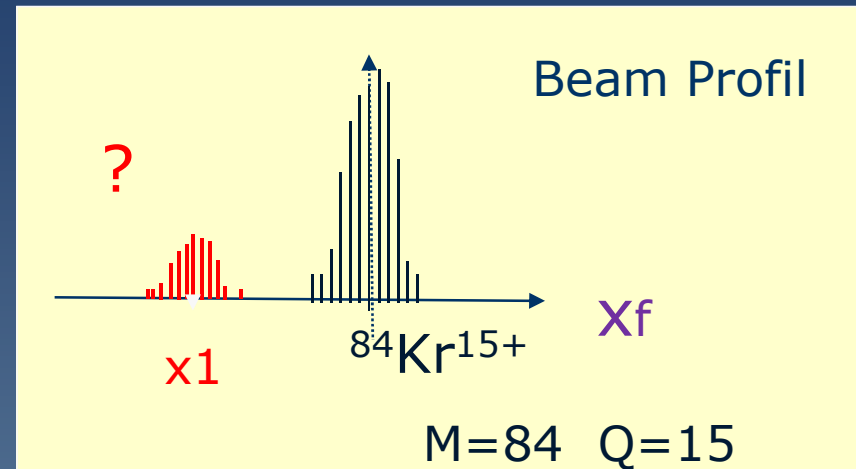
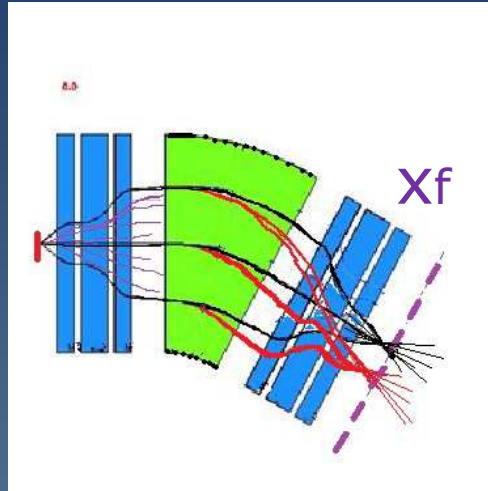
HORIZONTAL ANGLE

$$X' = dX/ds = \tan(\theta) \approx \theta$$

Magnetic Spectrometer :

A tool for identification

Suppose 2 ions beams



-Field measurement B

$$B_{p0} = B_{dipole} * R_{dipole}$$

-Position measurement (Xf = X1)

$$\delta = (B_{p1} - B_{p0}) / B_{p0} = X1 / R16$$

$$B_{p1} = B_{p0} (1 + X1 / R16)$$

If same velocity v

$$M1/Q1 \approx M0/Q0 (1 + X1 / R16)$$

The **R** matrix of spectrometer

: first order theory

A spectrometer

A) starts with a focus (on target)

B) End up with a focus ($R_{12}=R_{34}=0$)

C) The spectrometer is chromatic ($R_{16} \neq 0$)

typical matrix (8 coefficients)

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1 = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ - & - & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_0$$

$$l = v_0(t - t_0)$$

$$\delta = \frac{B\rho - B\rho_0}{B\rho_0}$$

R_{16} is called dispersion

R_{11} is called MAGNIFICATION



$$R_{11} = \Delta X_F / \Delta X_{\text{Target}}$$

Coordinates
At focal (detectors)

Coordinates
on target

$$x^F \approx \sum_{j=1 \dots 6} R_{1j} Z_j^0 = R_{11} \cdot x_0 + R_{12} \cdot x'_0 + R_{13} \cdot y_0 + R_{14} \cdot y'_0 + R_{15} \cdot l^0 + R_{16} \cdot \delta^0$$

Fragment Separators : 100-500 MeV/A

Reaction : in-flight fragmentation
(0° degree)

Goal :

- 1) **Primary beam suppression (Separator)**
- 2) Identification of particles
- 3) **purification** (selection of some reaction products)

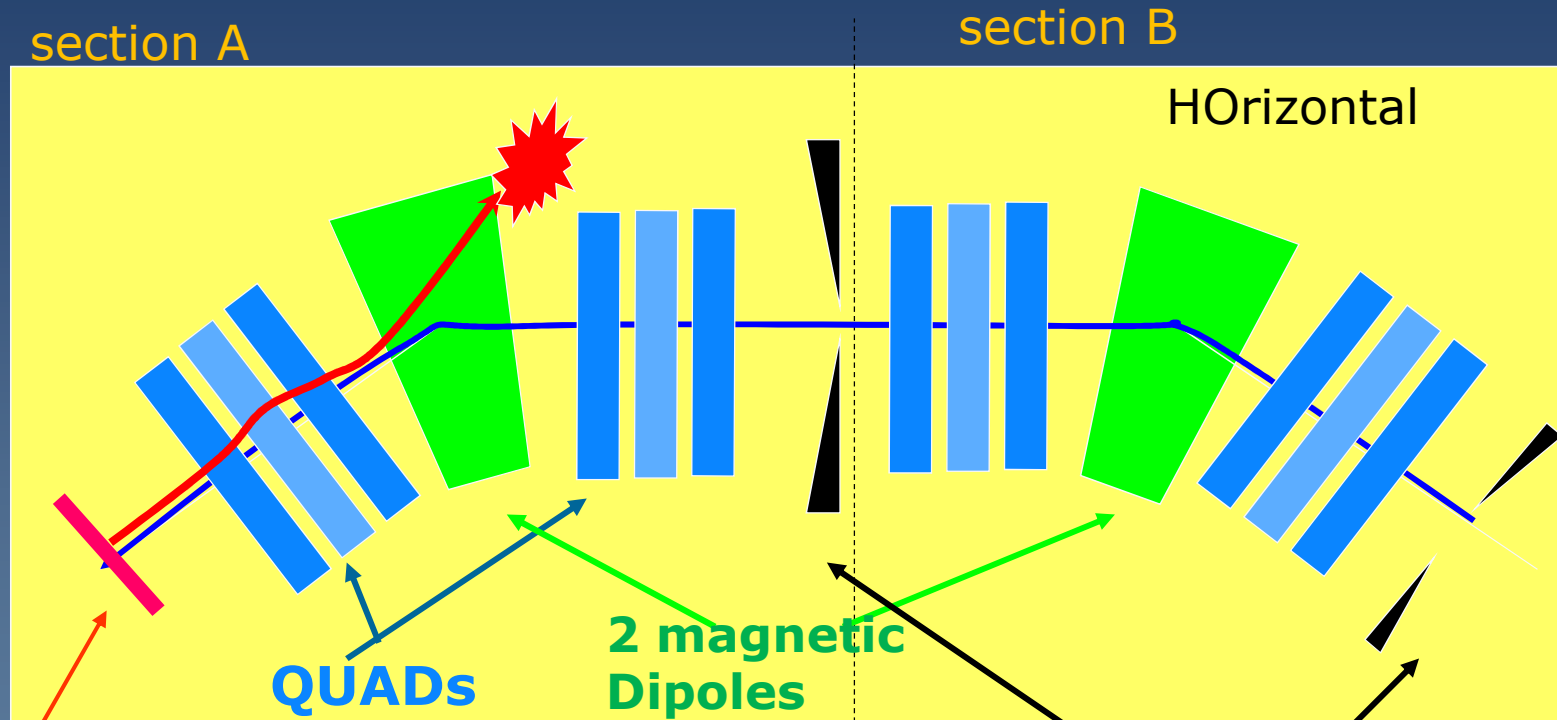


Fragment separator

2 symmetric sections :

« **ACHROMATIC** » MAGNETIC SPECTROMETER

From the top



Rotative Target

(high intensity primary beam)

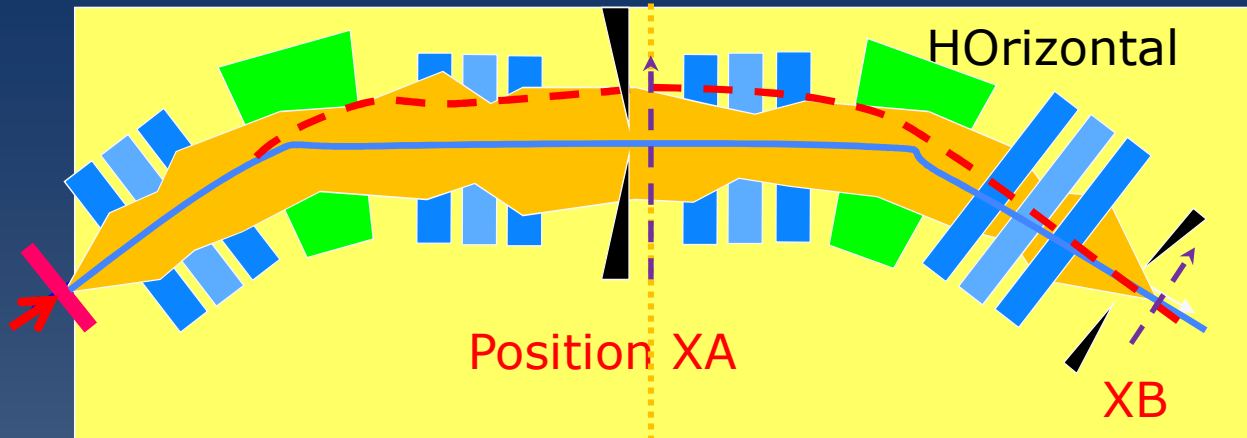
Nuclear fragmentation reaction

Selection slit

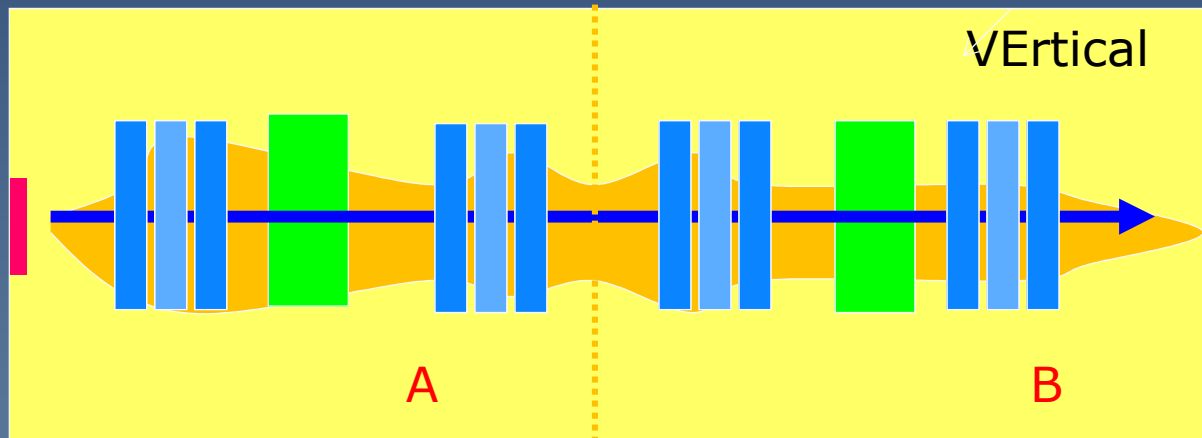
Total Transport matrix R

$$R = R_B \cdot R_A$$

Fragment separator : principle



Beam envelop HO



Beam envelop VE

Ion trajectory
 $X_A = F(B\rho)$
 $X_B \sim 0$
 « Achromatic »

$$R_{16}(A) \neq 0$$

$$R_{16}(B) \neq 0$$

$$R_{16}(A+B) = 0$$

A: 1st section

$B\rho$ selection

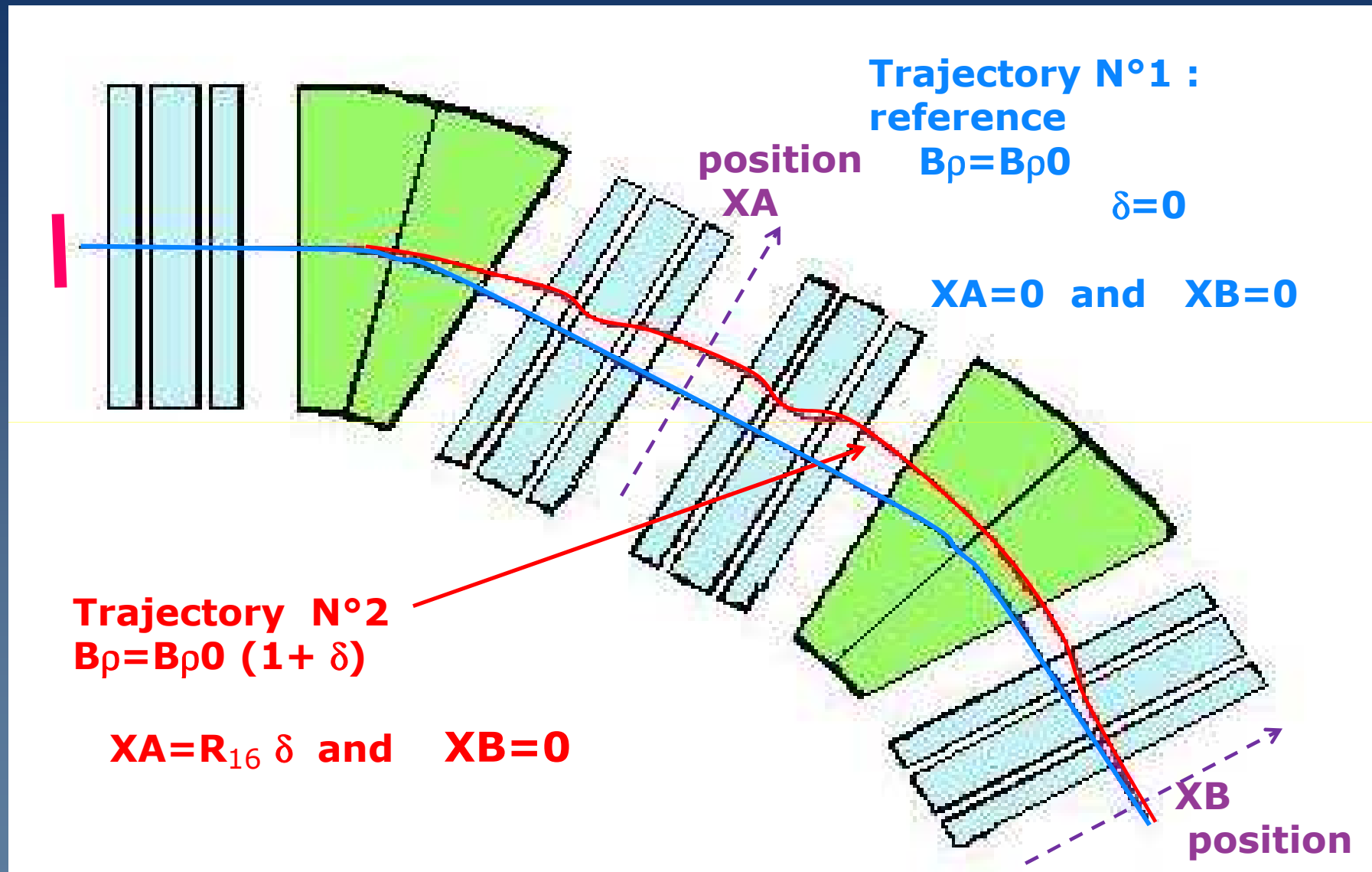
$$R_{16}(A) \neq 0$$

B: 2nd section

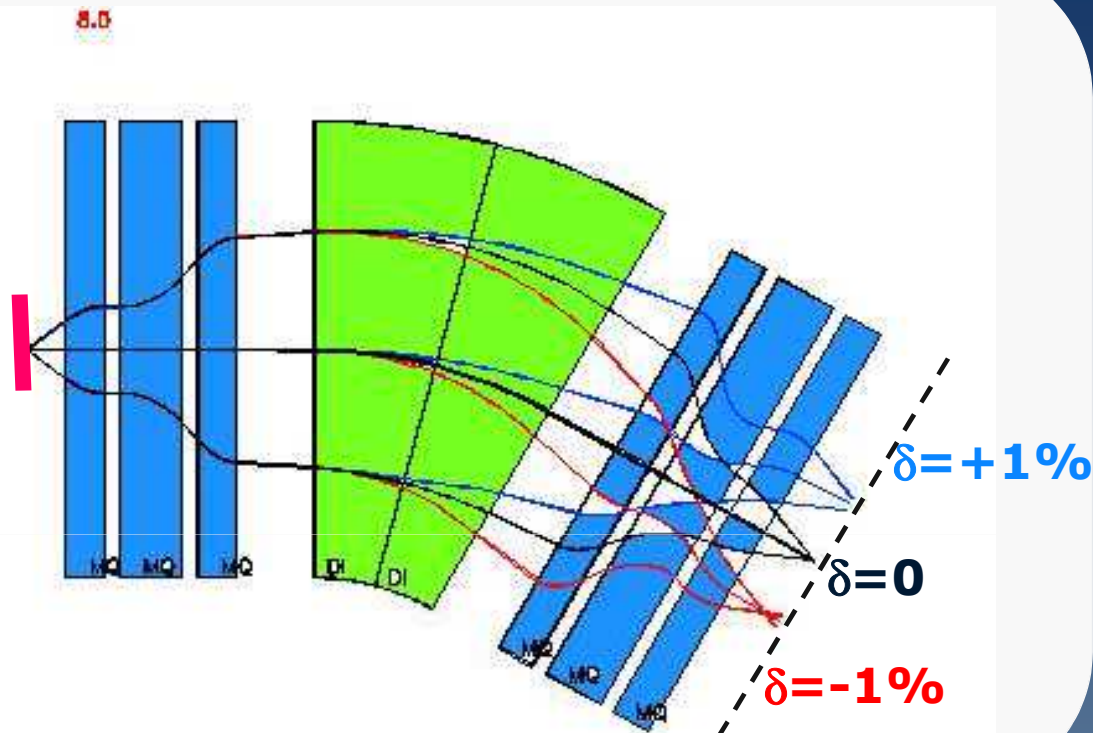
$B\rho$ compensation

$$R_{16}(A+B) = 0 \text{ (achromatic)}$$

2 Trajectories in a Fragments separator



Fragments separators : dispersiv section optics



Section A :

Focusing

$R_{12} = 0$ (Horizontal)

$R_{34} = 0$ (Vertical)

dispersion

$R_{16}(A) \neq 0$

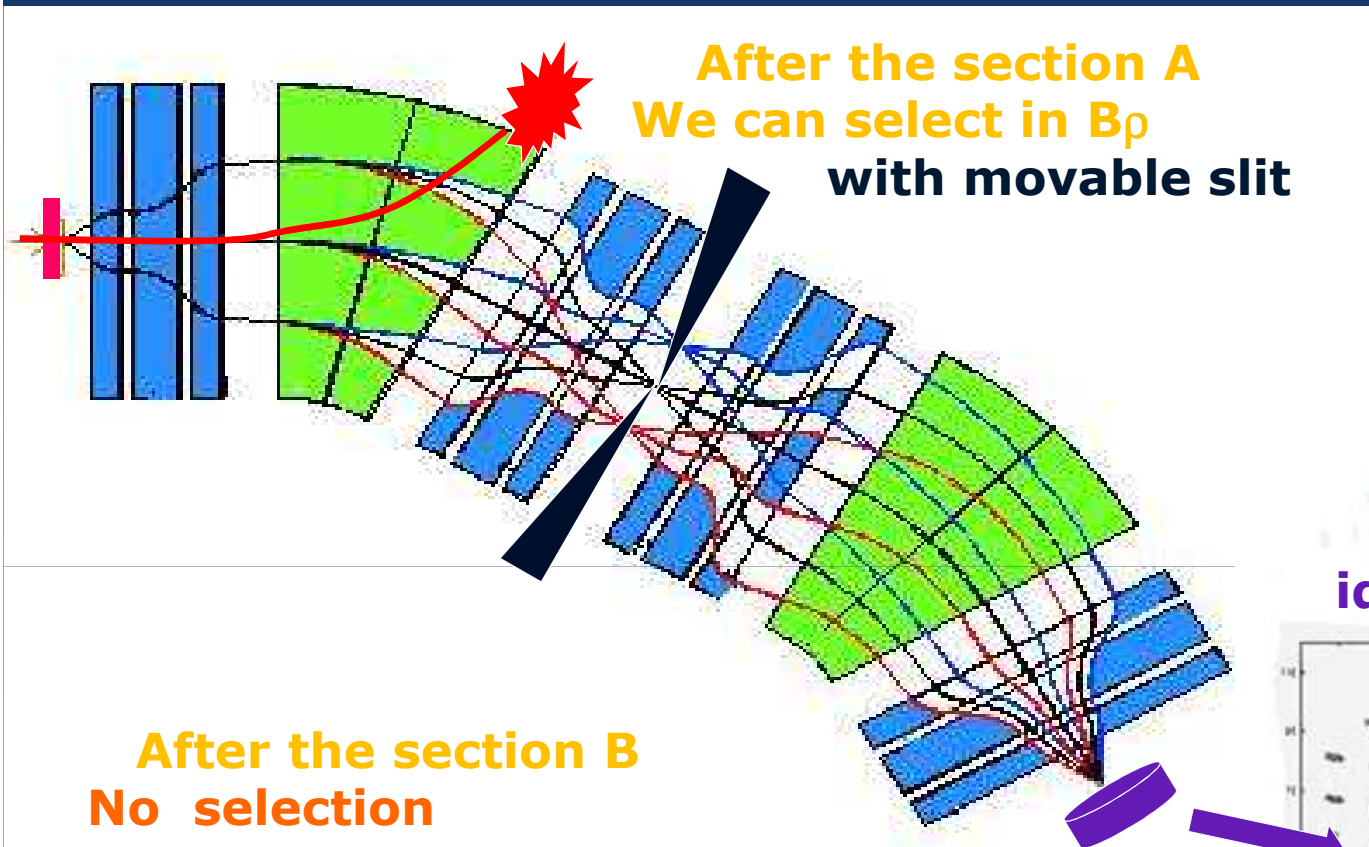
$$Rmatrix(A) = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Dispersiv focal plan

$$\delta = (B\rho - B\rho_0) / B\rho_0$$

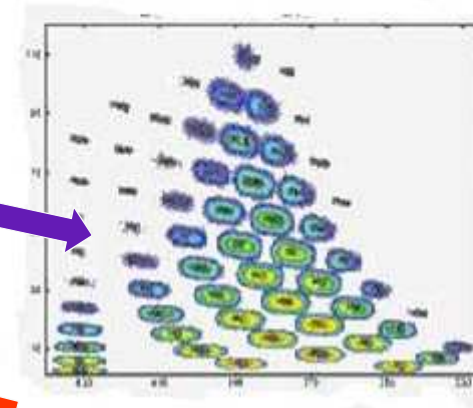
$$B\rho_0 = B_{dipole} \cdot R_{dipole}$$

1 Selection in Fragments separators is not sufficient



$B\rho$ Selection
Is not
good enough

$B\rho$ selection
identification



Primary beam is eliminated, but
Too Many isotopes ($\neq Z$)
produced by fragmentation
are transported up to the end

Magnetic separator with degrador increase the purification (Z dependance)

We consider 2 isobares ($A=34, Z=14$) ($A=34, Z=15$) with same $B\rho$

$B\rho$ selection is
independant from Z

$$B\rho = \gamma M v / Q$$

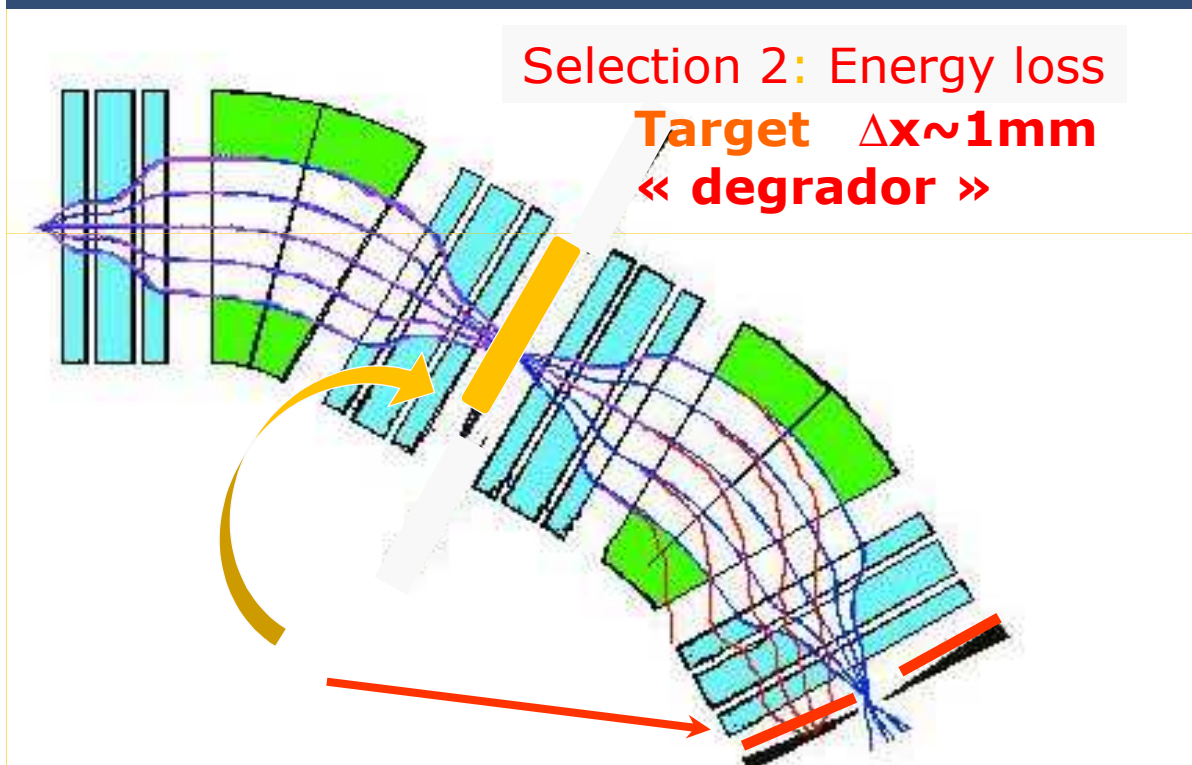
in a target Energy loss
is

« Z dependant »

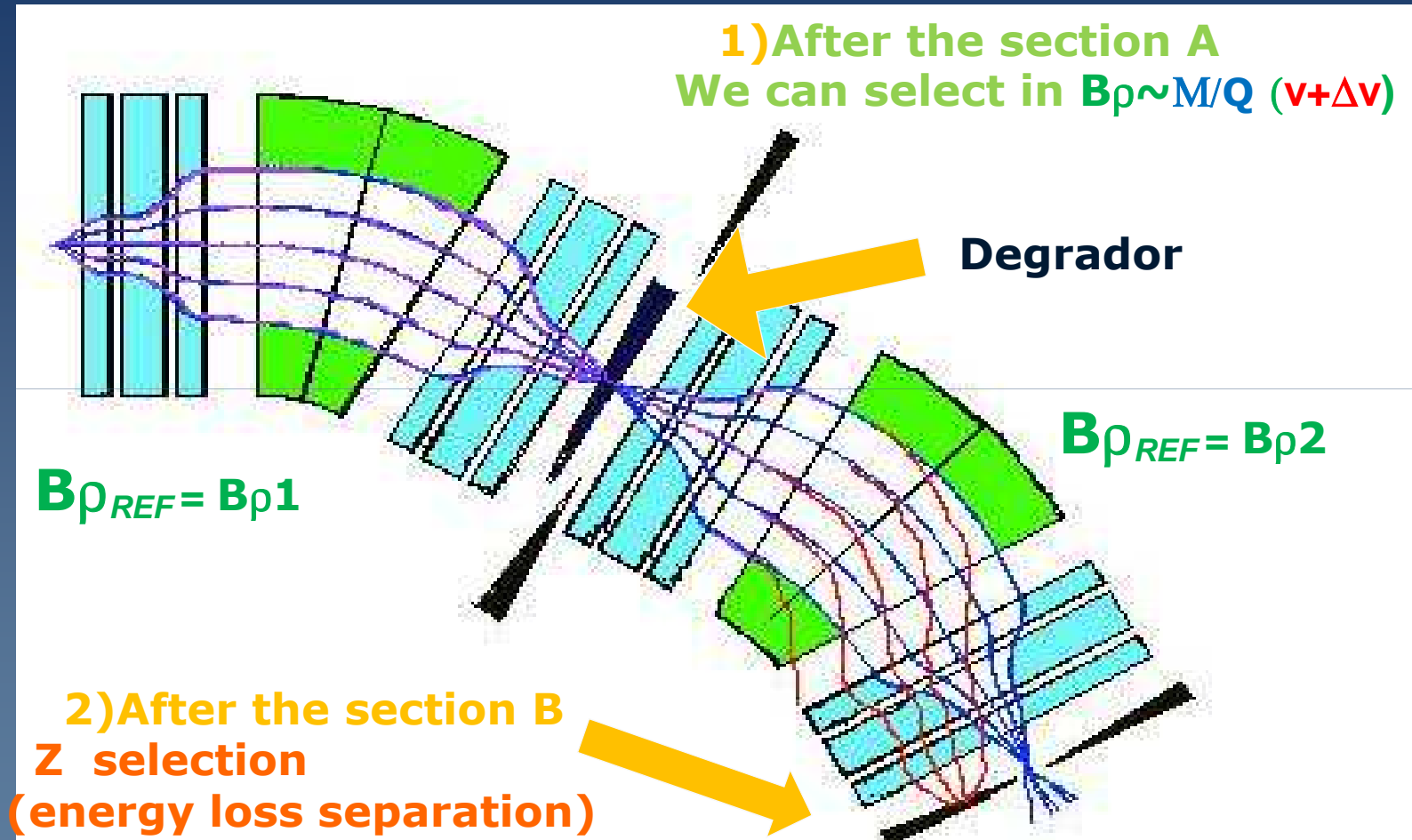
Bethe-Bloch
formula

$$\Delta E = k Z^2/A * \Delta x$$

42



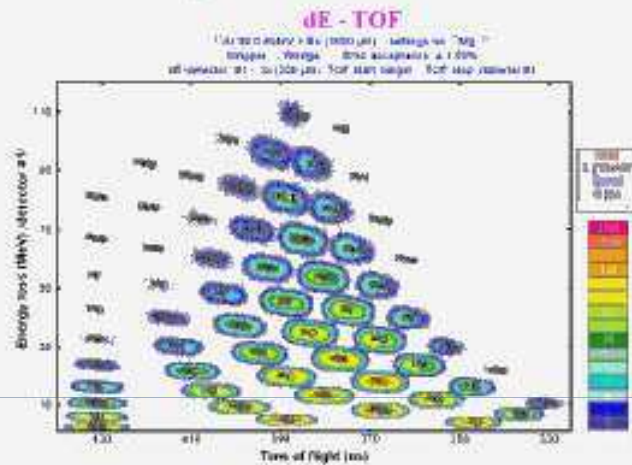
2 Selections in Fragments separators $B\rho + Z$ (degrador)



Selection in Fragments separators & identification

$B\rho$ selection

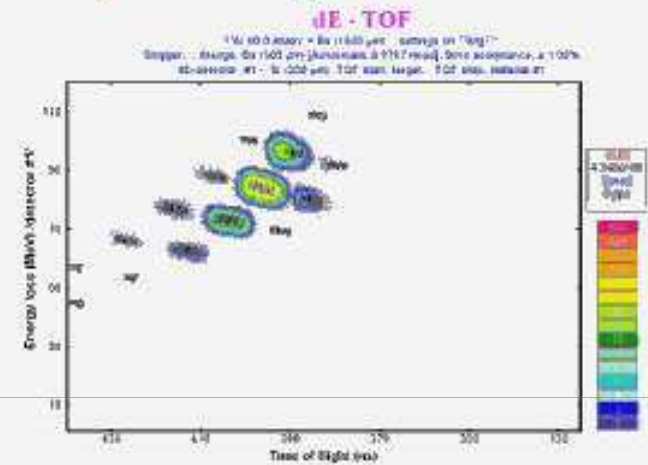
ΔE
 $\sim Z$



Time of Flight $\sim M/Q$

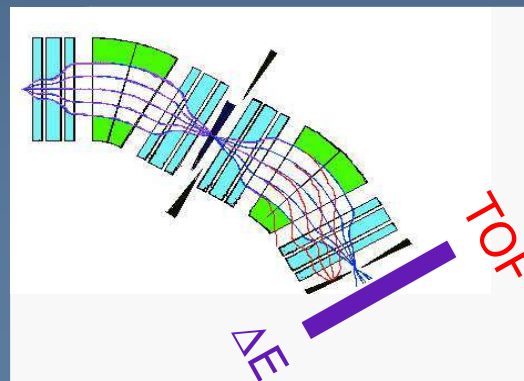
$B\rho$ + degrador selection

ΔE



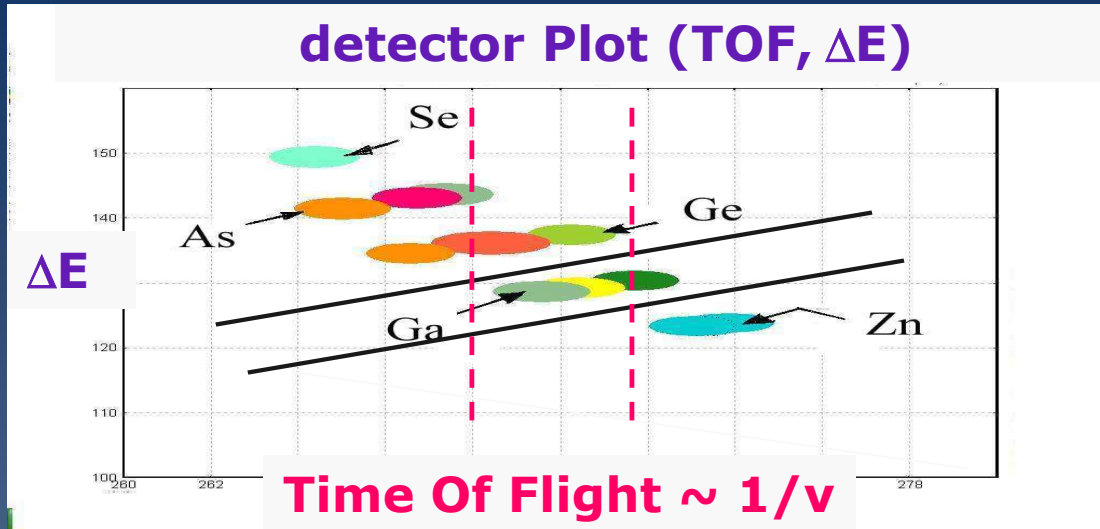
Time of Flight

Detector :
Thin Silicone



2 selections
Is much
better for
purity

Often, Isotopes are not well identified ($\Delta E, \text{TOF}$)



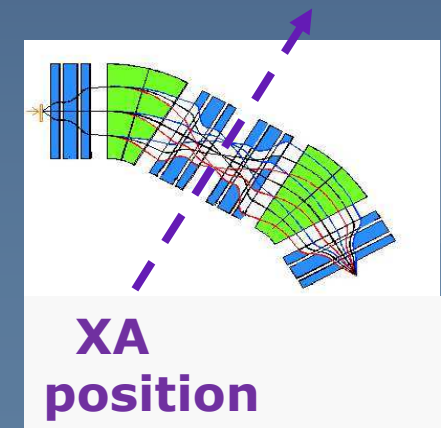
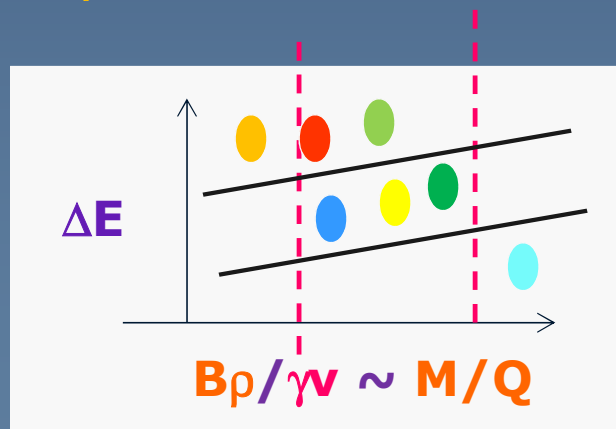
Isotopes
are **mixed** in TOF
Large velocity
distribution Δv

Solution : Measure X_A for each ion : $B\rho = B\rho_0 (1 + X_A/R16)$

The two measurements (TOF, $B\rho$) \Rightarrow give M/Q

$$v = \text{TOF} / L$$

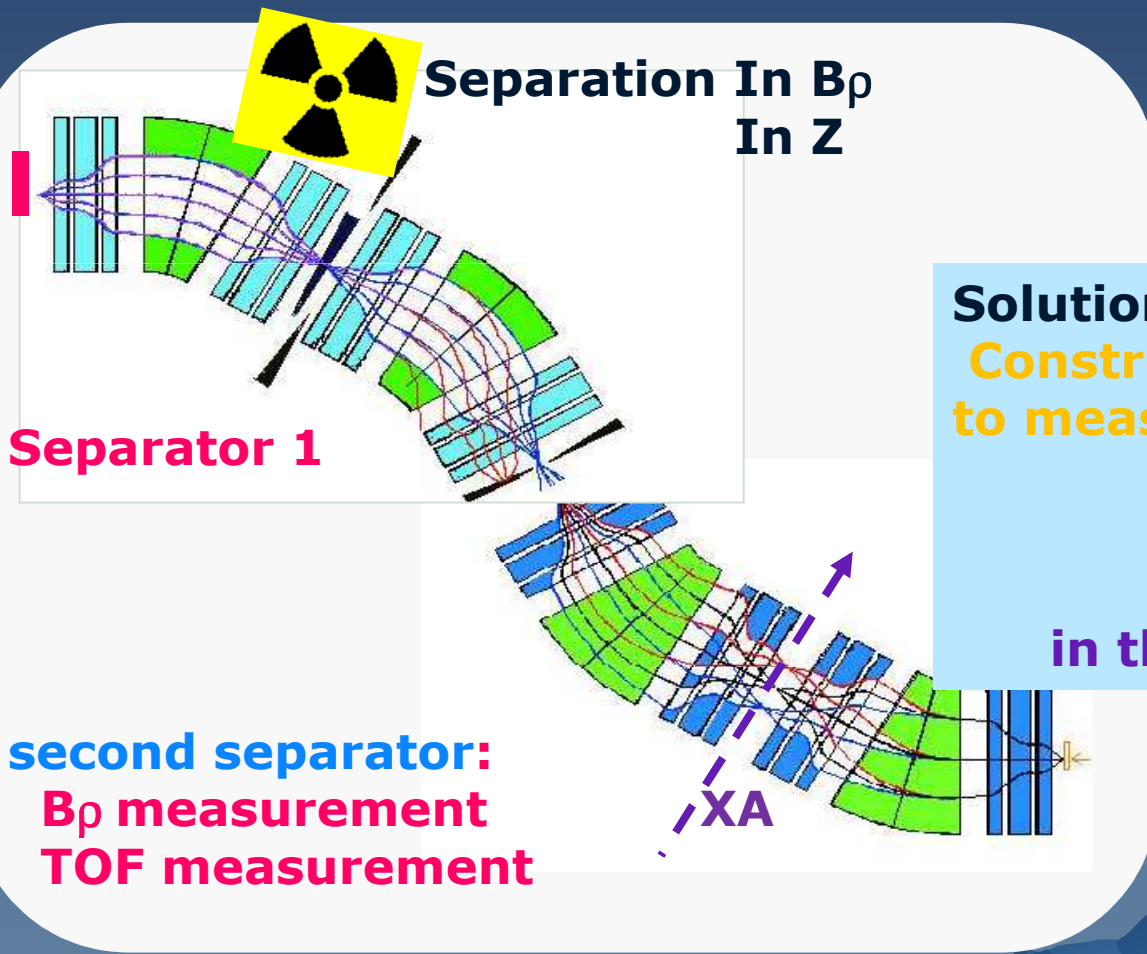
$$M/Q = B\rho / (v \cdot \gamma)$$



Isotopes are not well identified with ($\Delta E, \text{ToF}$)

Install a Detector position for XA : $B\rho = B\rho_0 (1 + XA/R16)$

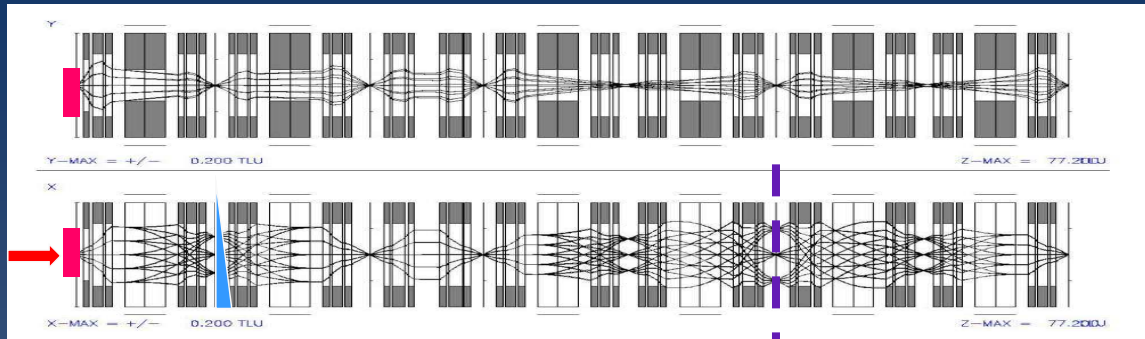
NOT POSSIBLE
(too much intensity before Z selection)



Solution chosen in BigRIPS :
Construct an additiv spectrometer
to measure $B\rho$ (€ !!)

Install the
position Detector
in the second separator

1 example :BIG RIPS (Riken)



Specifications

$$L=77\text{m}$$

$$B_{p\text{max}} = 7 \text{ Tm}$$

$$\Delta p/p = \pm 3\%$$

$$\Delta\theta = \Delta x' = \pm 50\text{mrad}$$

$$\Delta\phi = \Delta y' = \pm 60\text{mrad}$$

BigRIPS : Tandem (Two-stage) Separator

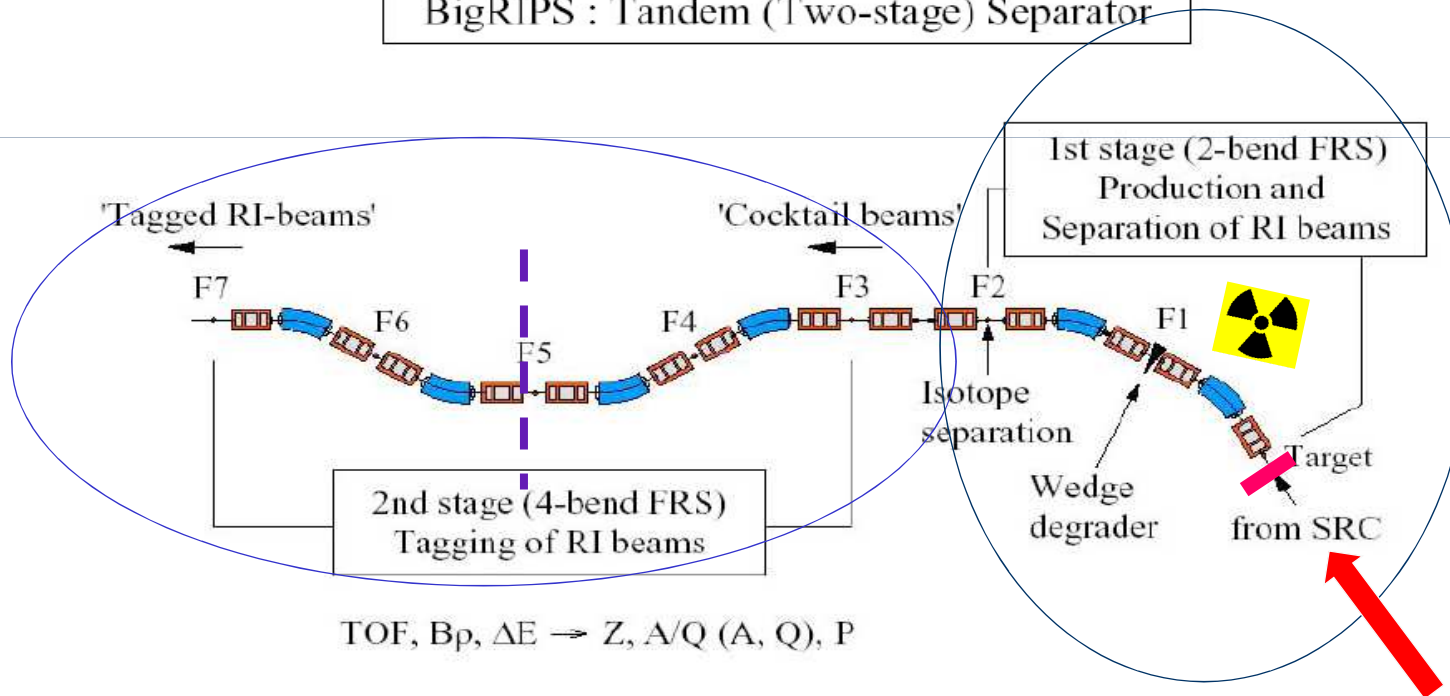


Fig. 2. A schematic diagram of the RI-beam tagging in the BigRIPS separator.

BIG RIPS (Riken) quads

Beam very rigid : $B\rho = \gamma m\mathbf{v}/Q = 7 \text{ T.m}$ (Beam **300MeV/A**)

with **high v** ! 

Super-ferric quadrupole triplet :

Very strong focusing : supraconducting coils (NbTi), with pole (Fe)

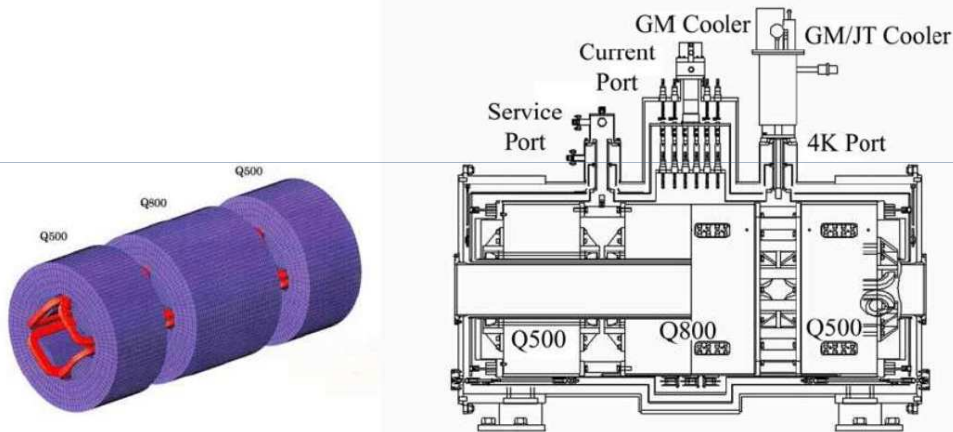


Figure 22: Schematic view of the RIKEN prototype quadrupole triplet (left side) and its installation into the cryostat (right side) [24].

- Supra-conducting coils (i very large, B close to saturation)
- Raperture very large = 0.1m ; Bpole-max# 2 Teslas
- GradientMax=2T/0.1m=20.T/m

Comparaison of the fragment separators

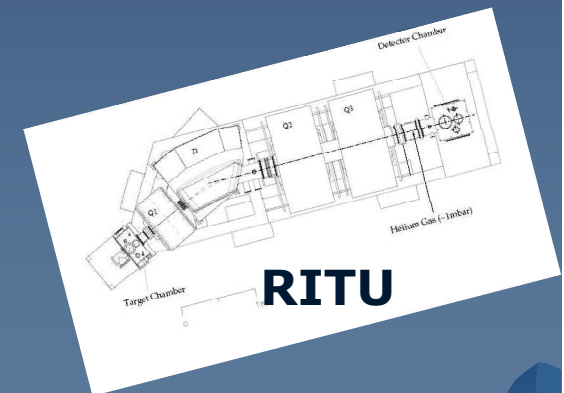
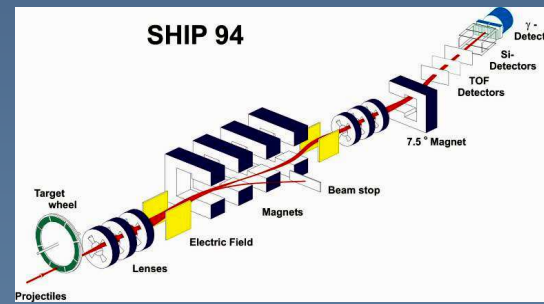
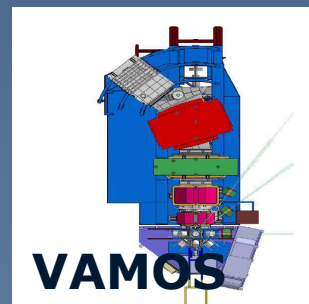
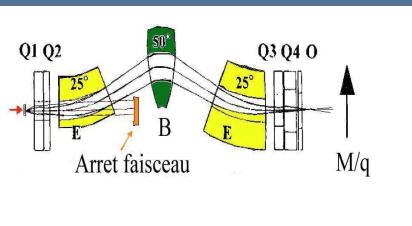
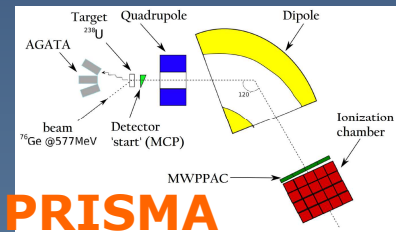
	Lise3 Ganil	FRS _{GSI} Mode1 or mode2	A1900 MSU //NSCL	BigRips Riken
Angular Acceptance	1.6 mstrd	0.32mstrd or 3.4 mstrd	8mstrd ±40* ±50mrad	10 mstrd ±50* ± 60mrad
B_ρ Acceptance	±2.5%	± 2.0%	± 3.0%	± 3.0%
R16 (m=cm/%) B_ρ Resolution Length	1.7 m 1/600 42 m	6.8m 1/1600 or1/160 69m	5.95m 1/2900 35m	3.3 m 1/3300 77 m
B_ρmax	4.3T.m //3.2T.m	18T.m or 8.6T.m	6.3 T.m	9. T.m
Comments	2 Dipôles + Wien filter	4 dipoles	4 dipoles	1 pre- separator (2 dipoles) + 1 separator (4 dipoles)

« Recoil » spectrometer : at low energy (1-10MeV/A)

Reactions : fusion-evaporation, transfer,..

Goals :

- 1) Very efficient primary beam suppression
- 2) Help identification



Many experimental problems => A Large variety of devices

Recoil spectrometer at low energy (1-10MeV/A)

- * Velocity filter

ship@GSI : 1MeV/A (heavy superheavy)

- * « RMS » (Recoil Mass Spectrometer)

(fusion evaporation,...)

- * Gas filled (Dubna, Darmstadt, Berkeley, Jyvaskyla, Riken)

1-5MeV/A Fusion evaporation
super-heavy production)

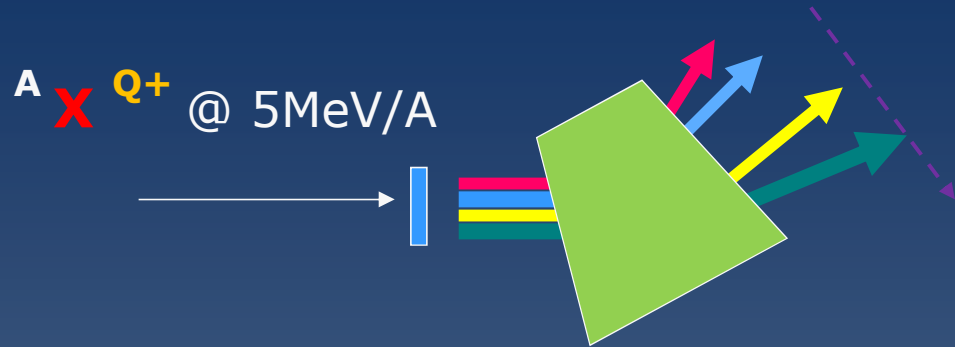
- * Large Acceptance & Ray tracing Spectrometer

Ganil (VAMOS) , Legnaro (Prisma), NSCL (S800)

(transfer reactions, fission,..)

....

1st problem at $E < 15$ MeV/A : charge state distributions



Atomic reactions (xray ,stripping)

#1mg/cm²

Q distribution connected to :

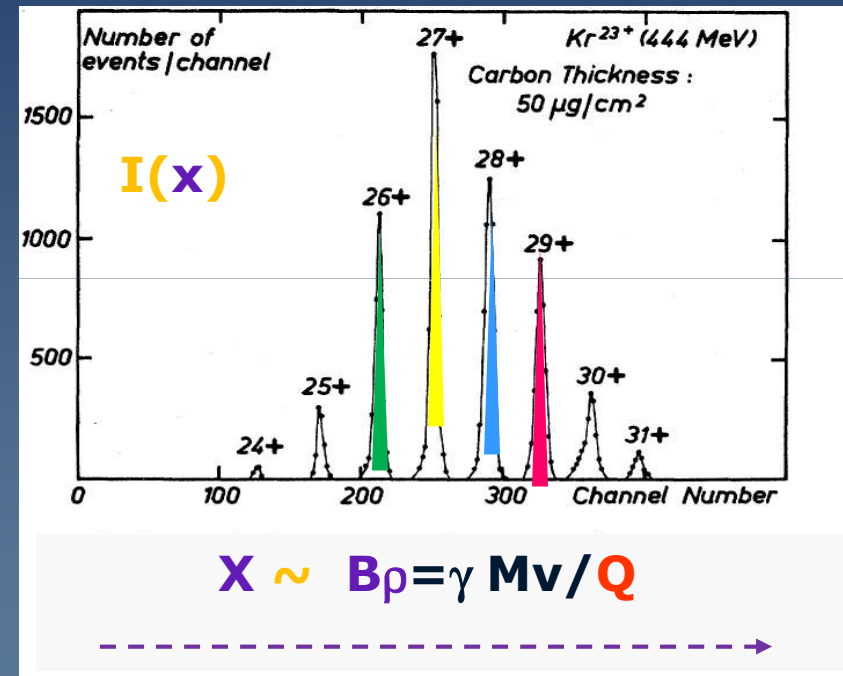
- Z target -Z projectile
- Exit energy -Target thickness

Many charge states

many sources of pollution of the focal plan detectors

(B_p is not a perfect for selection)

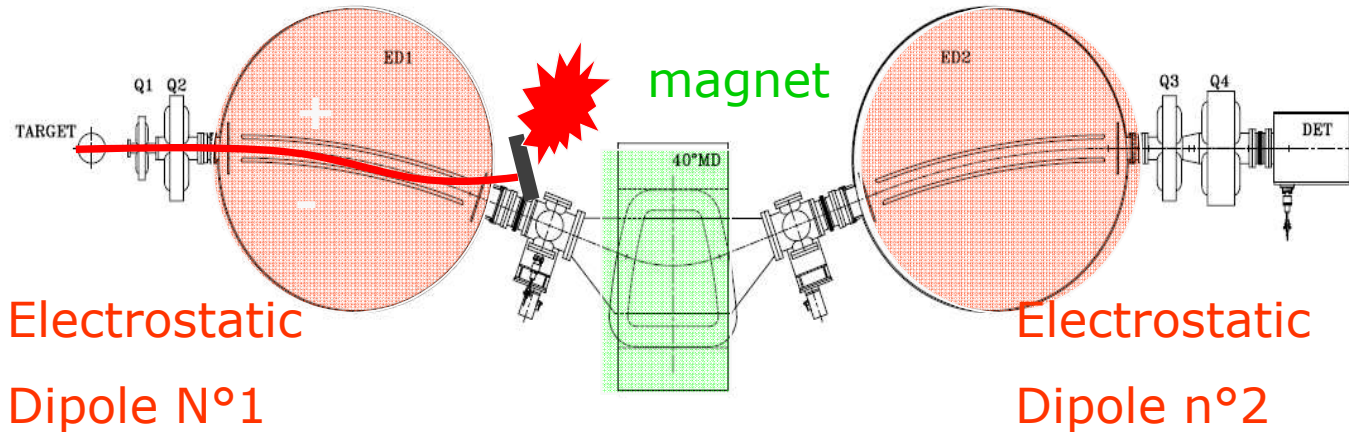
Stripping Probability



R_{ecoil} M_{ass} S_{eparator}

Emma
@Triumf

E# [1-3MeV/A] Length=12 m



$$\text{Magnetic selection} = B\rho = \gamma Mv/Q$$

$$\text{Electrostatic selection} = E\rho = \gamma Mv^2/Q$$

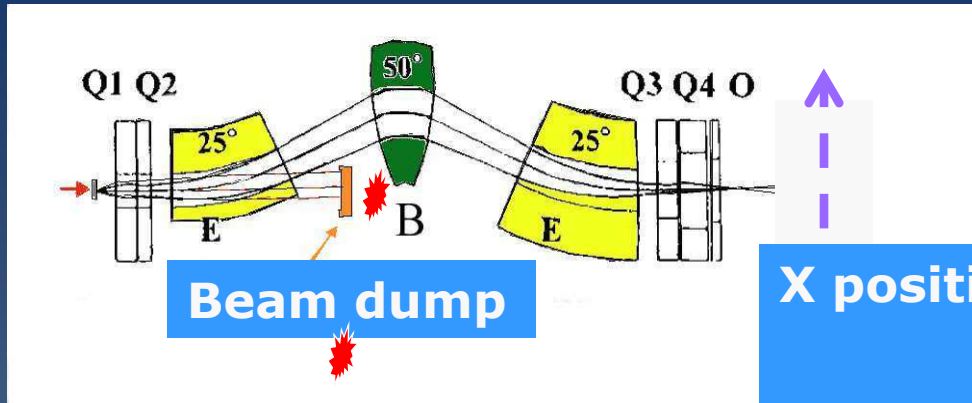
« RMS like Spectro. » Implemented in 6 different Laboratoires
(Oak ridge, Argonne, Legnaro, Jaeri, New Dehli, Vancouver) :

For **Fusion Reaction** : the Velocity is a good parameter for the selection

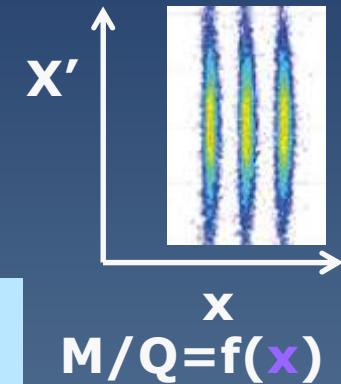
Electrostatic devices are efficient (but **sparking**)



RMS (Recoil « Mass » Spectrometer) : beam optics , M/Q dependance



Electrostatic selection
compensates
Magnetic selection



With E + B selection :
Achromatic Selection $X = f(M/Q, \text{velocity})$

Resolution : $R_{M/Q} = 1/300$

Resolution = $4 \sigma_{x_{\text{final}}} / R_{17}$

« R_{17} » is the « M/Q dispersion »

$$\vec{Z} = \begin{pmatrix} z1 \\ z2 \\ z3 \\ z4 \\ z5 \\ z6 \\ z7 \end{pmatrix} = \begin{pmatrix} x \\ x' = \frac{dx}{ds} \\ y \\ y' = \frac{dy}{ds} \\ l = v_0(T - T_0) \\ \delta = \frac{B\rho - B\rho_0}{B\rho_0} \\ \delta_m = \frac{M/Q - M_0/Q_0}{M_0/Q_0} \end{pmatrix} = \begin{pmatrix} \text{horizontal displacement} \\ \text{horizontal "angle"} \\ \text{vertical displacement} \\ \text{vertical angle} \\ \text{longitudinal difference} \\ \text{"(B\rho)" deviation} \\ \text{"mass(M/Q)" deviation} \end{pmatrix}$$

Gas filled separator for heavy ion

At low energy : too Many charge states

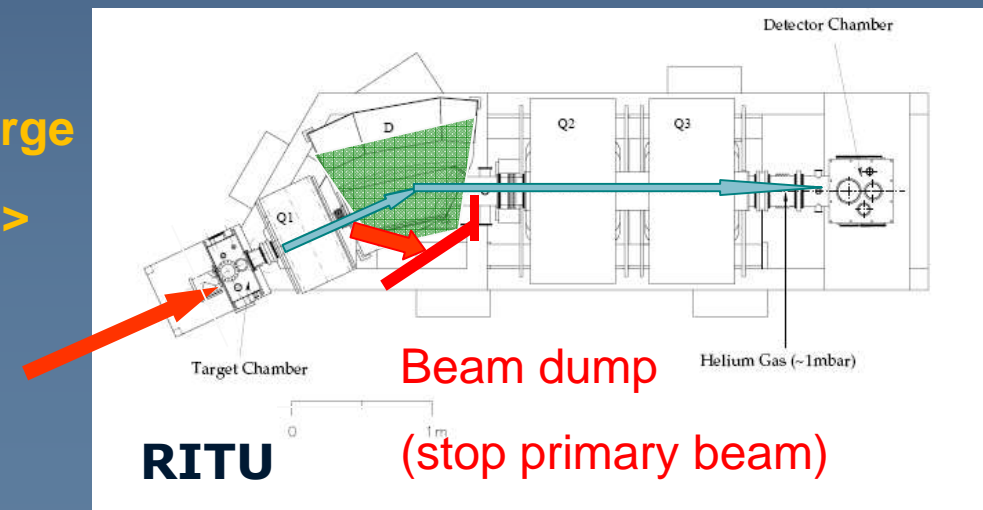
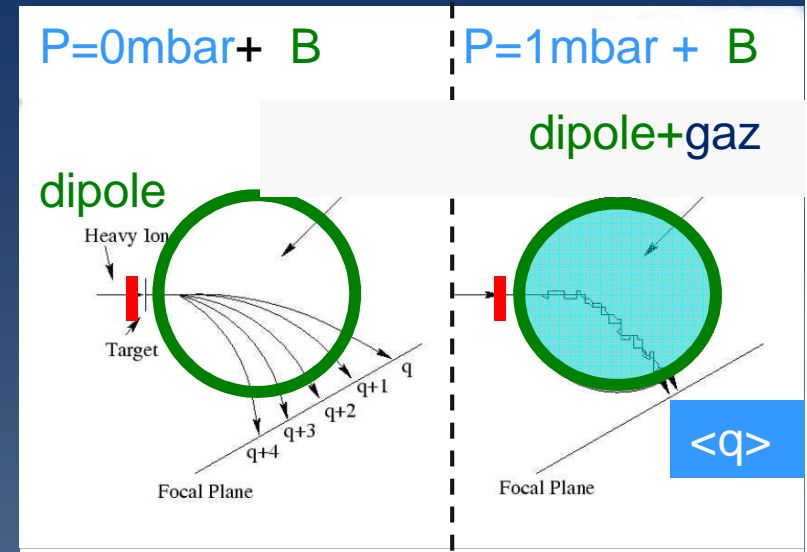
Beam charge are spilled over the focal plan

$$\langle q \rangle_{\text{gas}} \propto v Z^{1/3}$$

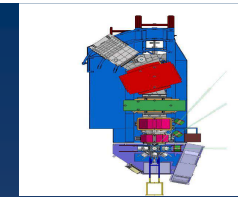
$$\langle B\rho \rangle = \frac{m}{\langle q \rangle} v \propto m Z^{-1/3}$$

In the gaz, the collisionS make the charge State oscillating around an average $\langle q \rangle$

« Charge focusing » + selection Mass
= good rejection



Large acceptance spectro



Optics is *non-linear* in x, x', y, y'

(Aberrations come with large angle x', y')

$$B\rho = B\rho_0 (1 + x/R_{16} + a x'^2 + b x^2 + c x^3 + \dots)$$

Vamos example :

In the focal plane, 7 quantities are measured :

$T, x_1, y_1, x_2, y_2, \Delta E, E$

T : Multi Wire PPAC

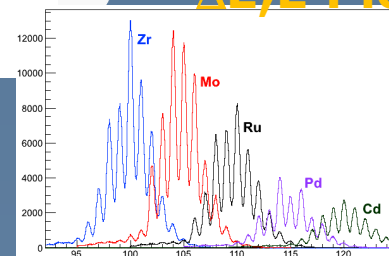
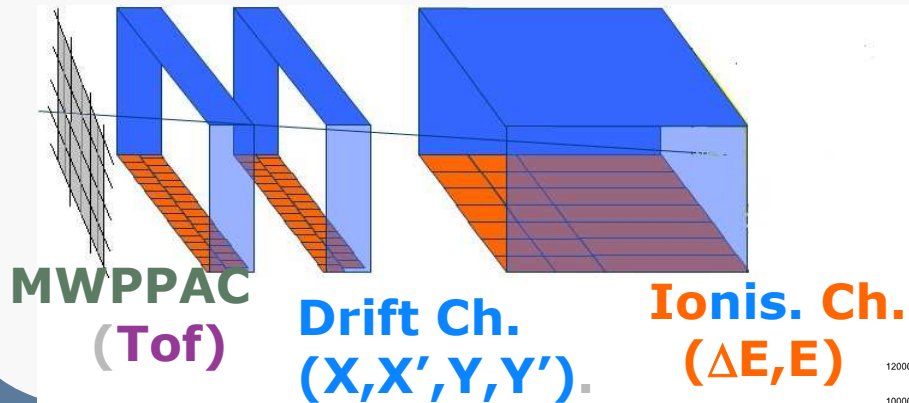
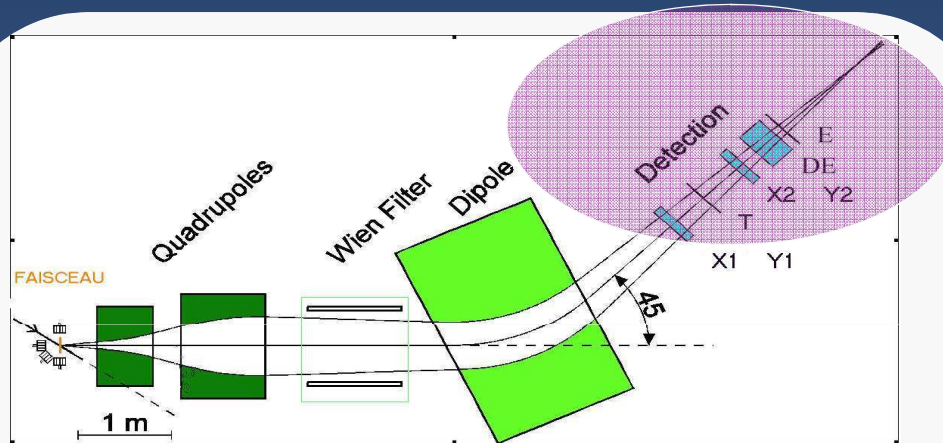
x_1, y_1

x_2, y_2 :

$$x' = (x_1 - x_2) / d = \tan(\theta)$$

$$y' = (y_1 - y_2) / d = \tan(\phi)$$

$\Delta E, E$: ionisation CHAMBER



SPECTROMETER TUNING AND DIAGNOSTICS

Tuning rely on - **B field** measurement
- Beam measurement

Beam Diagnostics : dedicated Robust detectors for
beam tuning

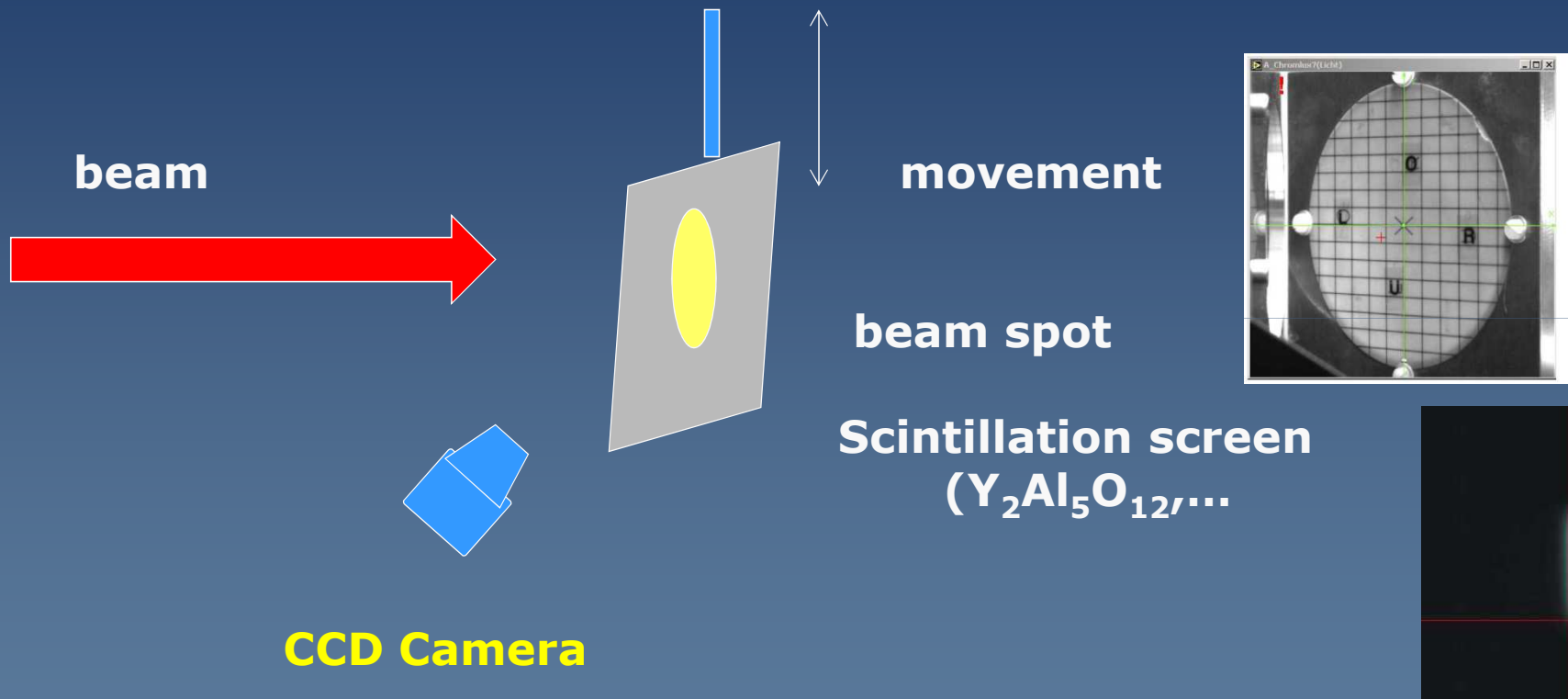
Statistical information on the beam (\bar{X} , σ_x , σ_T , $\langle I \rangle$...)

1rst step : check the primary beam

- profil measurement (alignement, focus)
- intensity check

SPECTROMETER TUNING

Beam diagnostics : scintillator screen



Relatively low cost , but

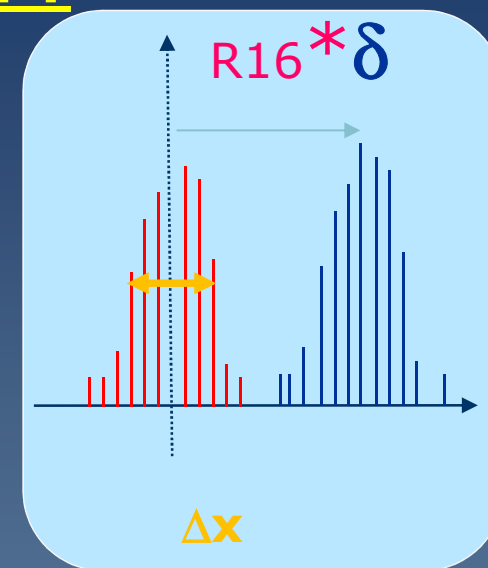
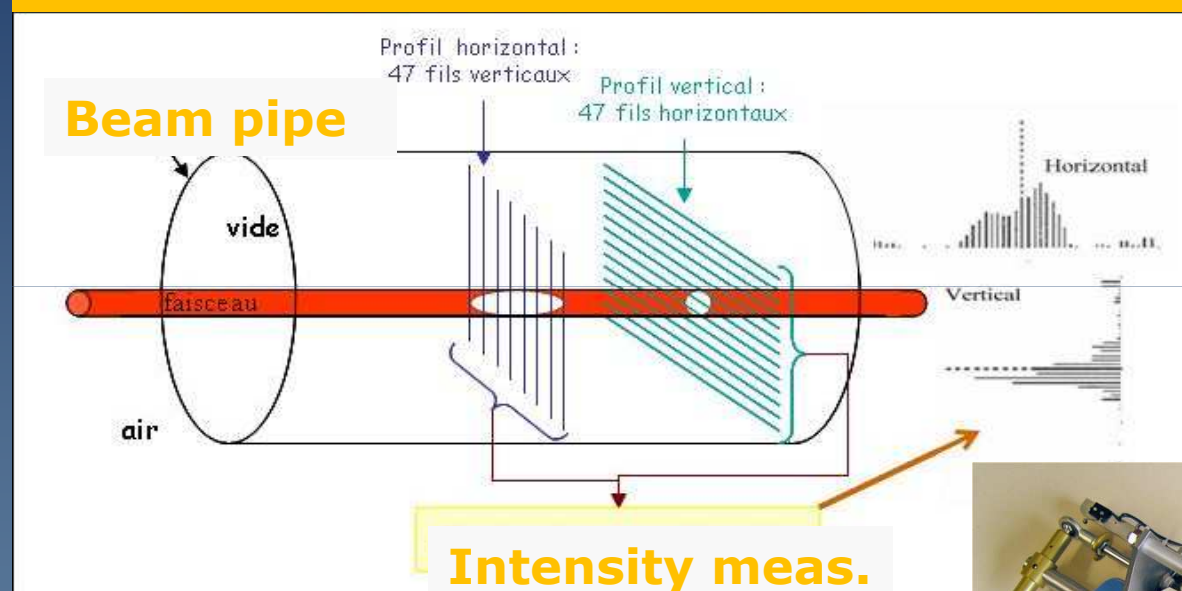
- only 1 profil measurement
- not very precise

SPECTROMETER TUNING

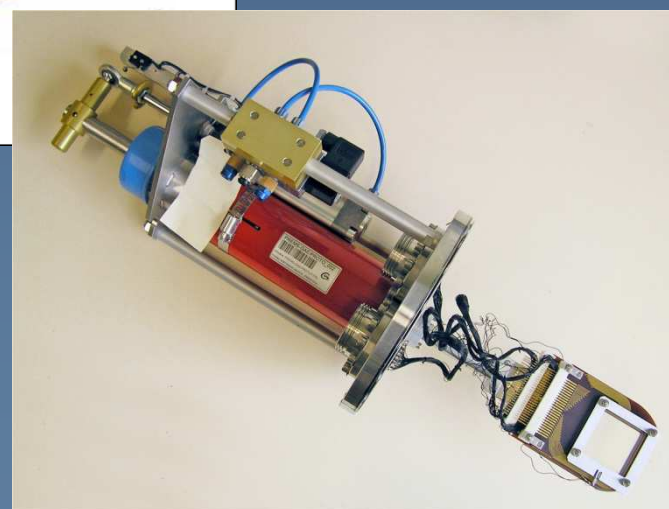
Beam diagnostics : **profil monitor**

Reconstruct the beam intensity in X and Y

Profil monitor : HORIZ. and VERT. wire



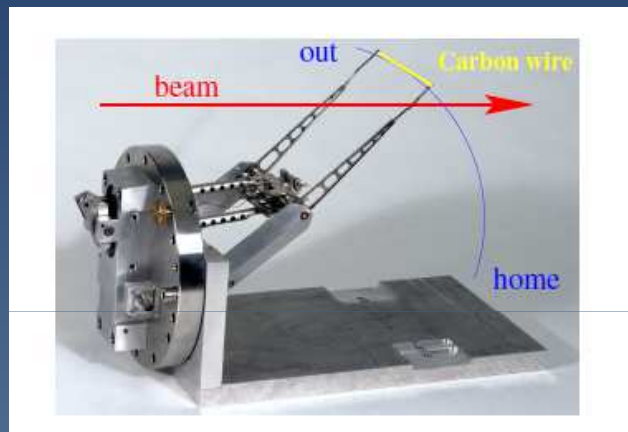
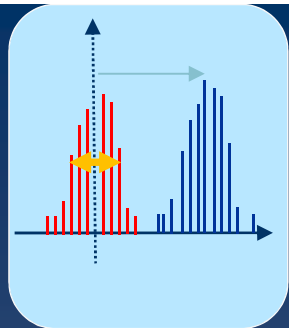
Usefull for **beam alignment**
focusing check
R16 measurement



SPECTROMETER TUNING

Many Profil monitors

for different beam intensities



Rotating wire
i# 10^{12-14} pps
(Cern)



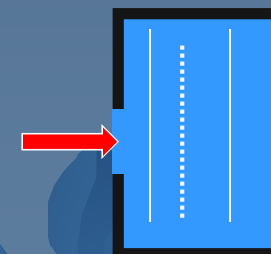
Wires
i# 10^9-11 pps

(Ganil)

« Gas Profil »
i# 10^3-7 pps

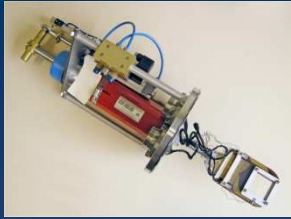


Gas ArCO₂ +HV



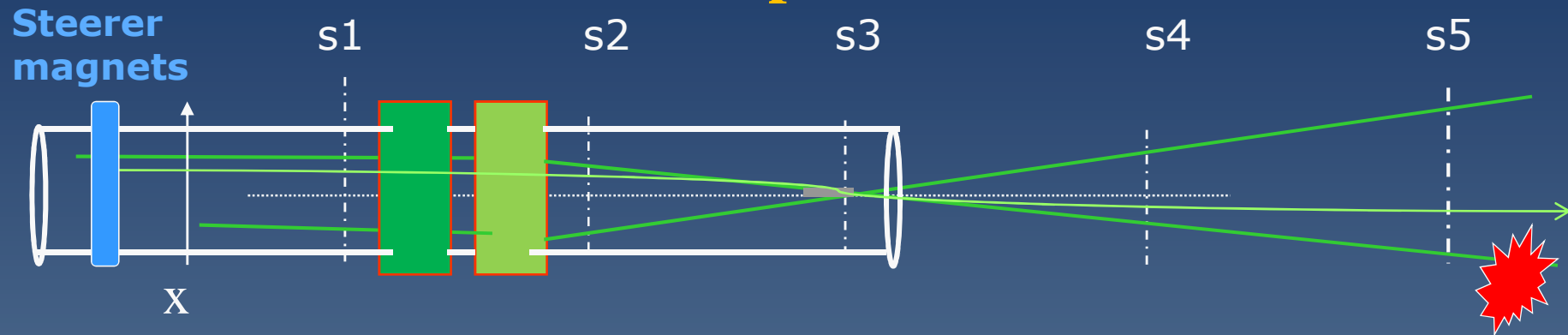
Proportional counter

Specific technologies
adapted for \neq (intensities, Energies)

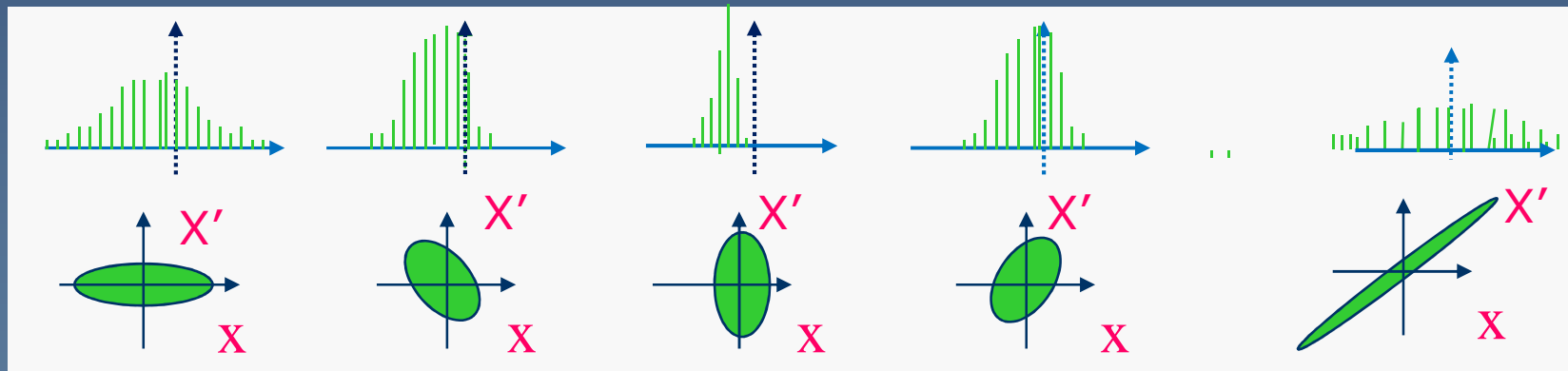


TUNING

Checking size and alignment with profil monitor



Beam
Profil
monitor



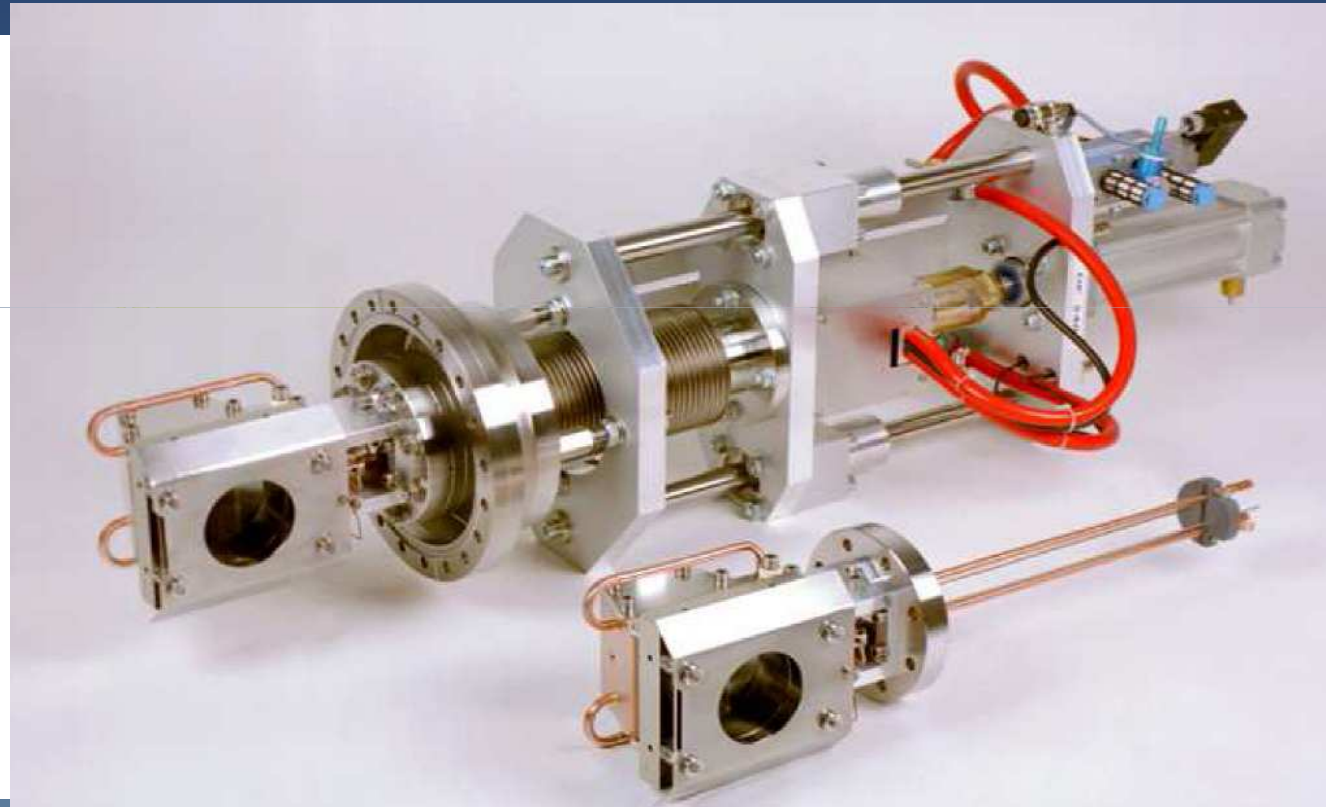
ellipsoid area = $\pi \Delta x \cdot \Delta x' = \text{Emittance}$

Emittance = constant if Energy = constant

SPECTROMETER TUNING : check the intensity

Beam diagnostics : **Faraday cup**

Intensity measurement



Particle per second

$$N_{pps} = IA/Qe$$

$$= I_{\mu A} 10^6 / [Q 1.6 \cdot 10^{-19}]$$

SPECTROMETER TUNING

Adjusting field in dipoles and quadrupoles

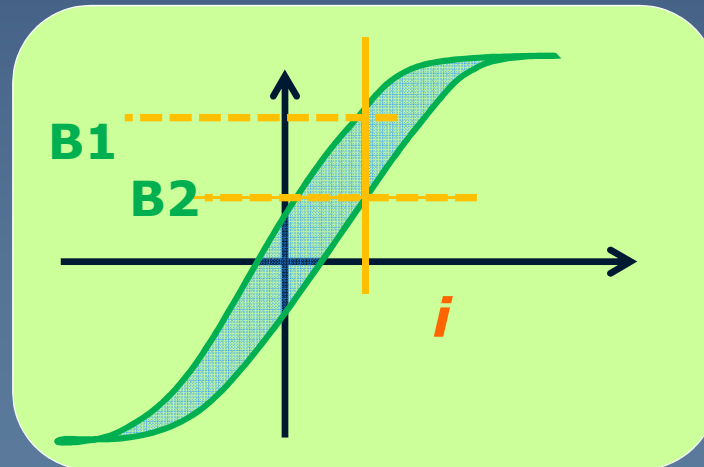
For adjusting dipole field (B) or quad Gradient (G)
adjust i in the coils

$B=B(i)$
and Gradient = $G(i)$ are given by the constructor

PROBLEM : hysteresis curve

$B=B(i)$

The current i gives
an information
About B **but**



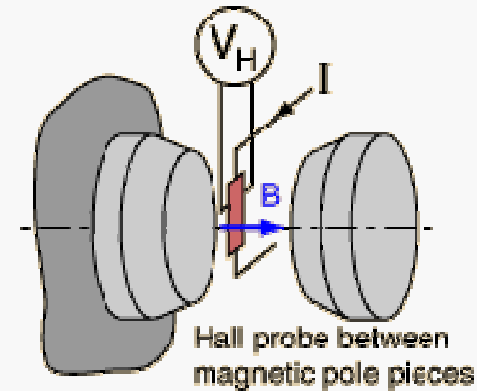
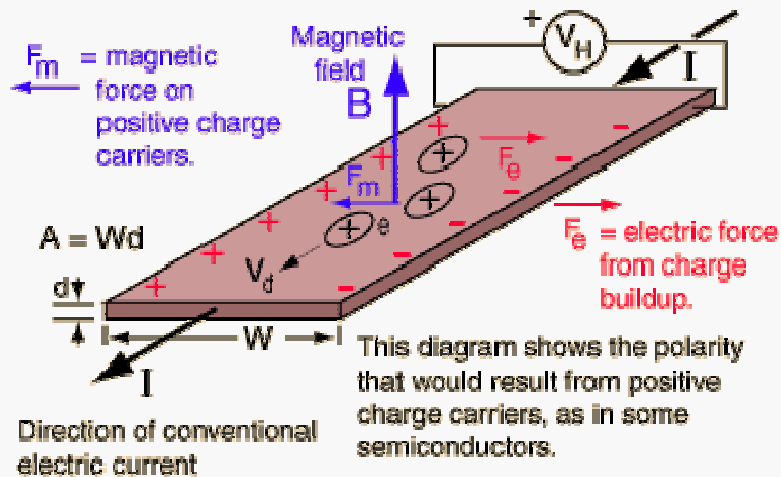
but for one i ,
2 possibilities
 $B1$ and $B2$

Solutions

- Raise the current to **i_{max}** , then get down & adjust i :
for reproducibility
- Measure B with Hall probe or NMR probes (dipole)

SPECTROMETER TUNING

Hall Probes : measuring field in dipole



The polarity of the Hall voltage for a copper probe shows that electrons are the charge carriers.

Hyperphysics (gsu)

LOW COST , But not very precise

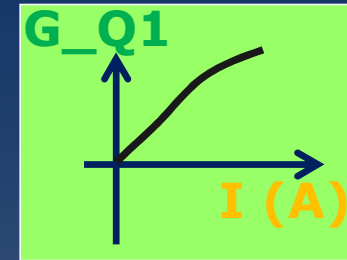
NMR probes are more precise (Resolution = 10^{-5})

Spectrometer tuning : before experiments

Quad Gradient : $B_y = G(\text{icoil}) \cdot X$

$G_{Q1}(\text{icoil})$ given by constructor

$B_{\text{dipole}}(\text{icoil})$ measured on test bench

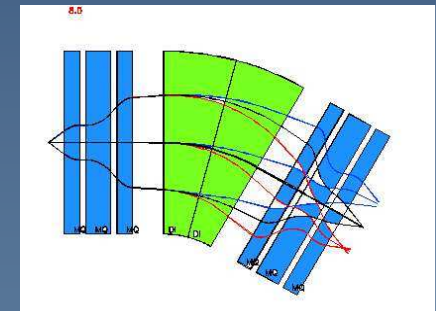


Rdipole has to be known (curvature of the ideal trajectory $R_{\text{dipole}} = L/\theta$)

Beam optics (Design step)

« beam optics » (quad setting for focusing on detectors, target..)

Compute G_{Q1} , G_{Q2} ... For $B_{\text{pref}} = 1$ Tm (simulation)
to get the right focus on the detectors



Experiment preparation : step 0

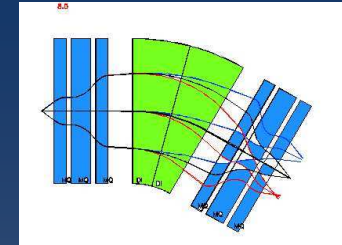
-Evaluate the B_p of the desired Ion beam

-Which beam optics to be used ? (detector location?,...)

Spectrometer tuning : **during the** experiment

Step 1 : Check the beam alignment

Step 2 : check the focus on target



Step 3: set the quad & dipole magnets (icoils)

With the « **Control command software** »

Select

the quad setting :« the beam optics » (focus on your detector)

the Rigidity $B\rho_0$ of the desired ions : $B = B\rho_0 / R$

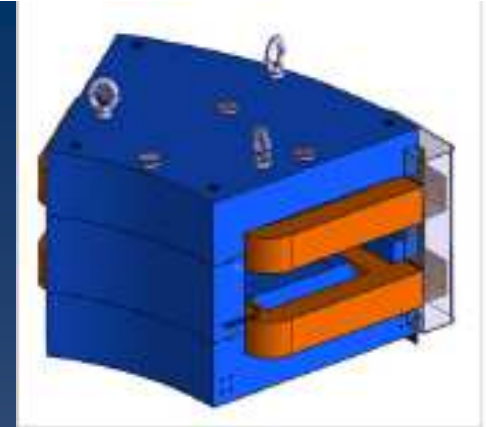
The Software Computes the fields by scaling:

$$G_{Q1} = G_{Q1} * B\rho_0 / B\rho_{Ref} \quad (\text{beam optics } N^\circ xxx)$$

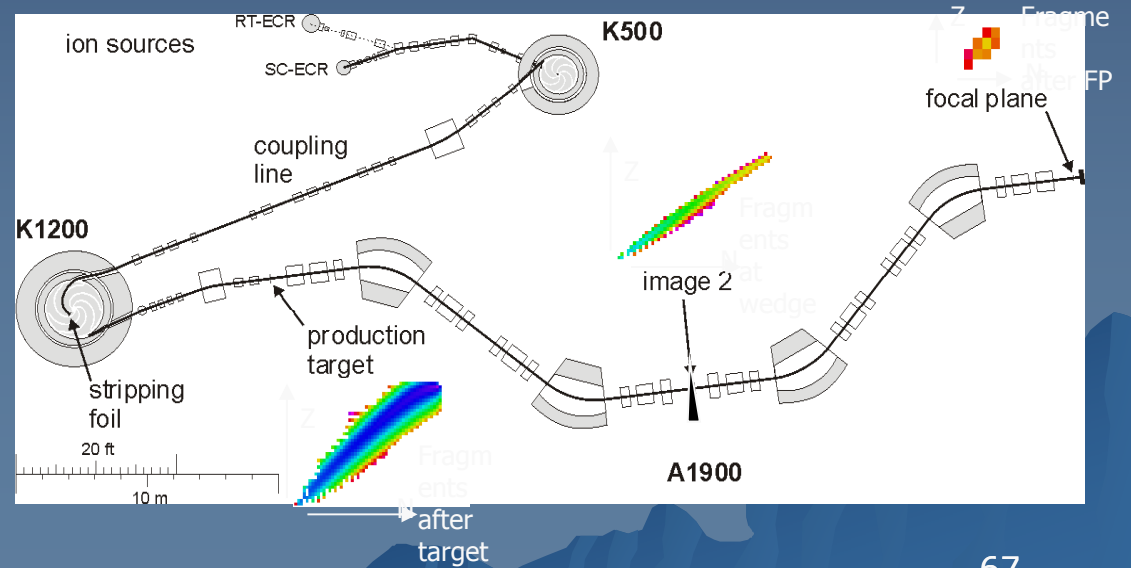
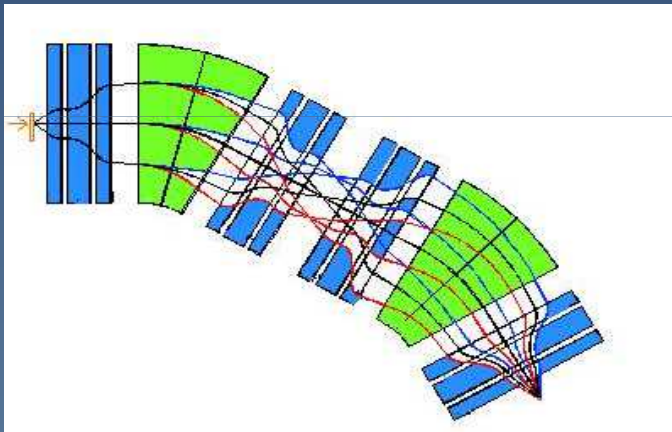
$$B_{dipole} = B\rho_0 / R_{dipole}$$

The **software computes the currents *icoils*** For the quads & dipoles coils

then, **check the dipole field B_{dipole}** with probes



End



References:

The historical paper for fragment separators:

[1] R.Anne, D.Bazin, A.C.Mueller, J.C.Jacmart and M.Langevin, "The achromatic spectrometer LISE at GANIL", NIM A257 (1987) 215-232.

More on wedge (degrador)

[1] H.Geissel, G.Munzenberg, K.Riisager, "Secondary exotic nuclear beams", Annu. Rev. Nucl. Part. Sci. 45 (1995) 163-203.

Interesting details in : Kubo et Al, Bigrips NIM

Large acceptance spectrometer :

M. Rejmund, Nucl. Instr. and Meth. A (2011)

Beam diagnostics

Peter Forck : Joint University Accelerator School 2006

Part of this lecture inspired by

B. Jacquot : JoliotCurie school 2008

Many Thanks to Catherina Michelagnoli,...

to my colleagues from Riken, GSI, NSCL, Jyvaskyla,
Triumf, Dubna, Legnaro, and Ganil

Back-up slides

- More on matrices
- Real Performance of a set-up (spectrometer+detector)
- How to optimise beam quality& Acceptance
- The Lise fragment separator & the wien filter
- Why the degrador thickness (Wedge) is not constant in a fragment separator ?
- Non linear effect in optical systems
- Examples

More on Transport Matrices:

Rmatrix for a straight section L (drift)

Particle Evolution in **drift** length between s_1 & s_2 :

$$x=x(s) \quad y=y(s) \text{ ???????}$$

$$x_2 = x_1 + \tan(\theta_1)(s_1 - s_2)$$

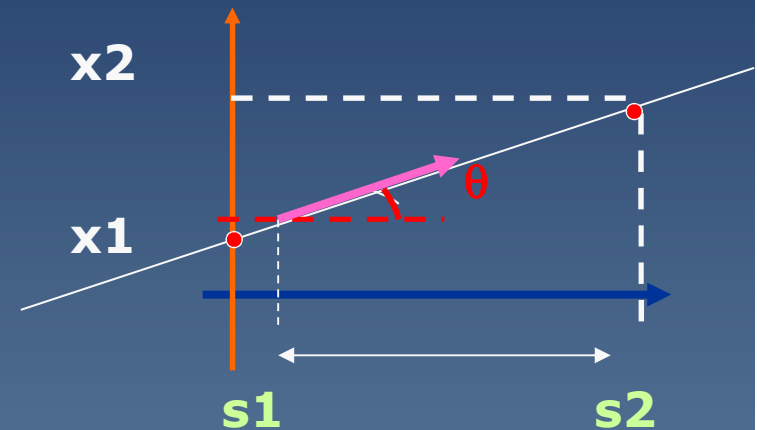
$$\theta_1 = \theta_2$$

$$y_2 = y_1 + \tan(\varphi_1)(s_1 - s_2)$$

$$\varphi_1 = \varphi_2$$

nota: $\tan(\theta_1) = dx_1/ds = x_1'$

and $(s_2 - s_1) = L$



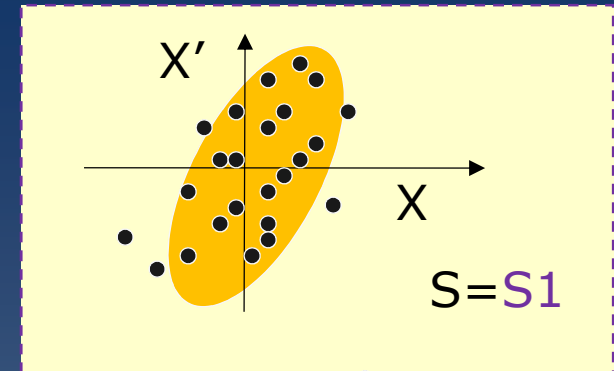
$$\begin{pmatrix} x_2 \\ x_2' \\ y_2 \\ y_2' \end{pmatrix} = \begin{pmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix}$$

$$R_{d1} = \begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The **beam** : N particles in a 6D ellipsoid

$$\sigma_x^2 = \sigma_{xx} = \sigma_{11} = \frac{1}{N} \sum_{\alpha=1, \dots, N} (x_\alpha - \bar{x}) \cdot (x_\alpha - \bar{x})$$

$$\sigma_{xx'} = \sigma_{12} = \frac{1}{N} \sum_{\alpha=1, \dots, N} (x_\alpha - \bar{x}) \cdot (x'_\alpha - \bar{x}')$$



1) σ_{ij} is a statistical definition of the beam

2) An optical code

Computes σ_{Final} with the R matrix at the end of the spectrometer

$$\sigma_{final} = R^T \cdot \sigma \cdot R$$



Done by simulation code

R Matrix allows the simulation

- a) -of the beam size $\sigma(s)$
- b) -of one trajectory $Z(s)$

Real performance of a (spectro+detector) depend on the experiment

efficiency = $\epsilon_{\text{detector}} \times \text{Transmission}_{\text{spectro}}$

Rejection = primary beam on target /
primary particle on final detector

Selectivity = ability to see the desired events in the background
(coincidence, identification)

Sensitivity = the smallest measurable cross section

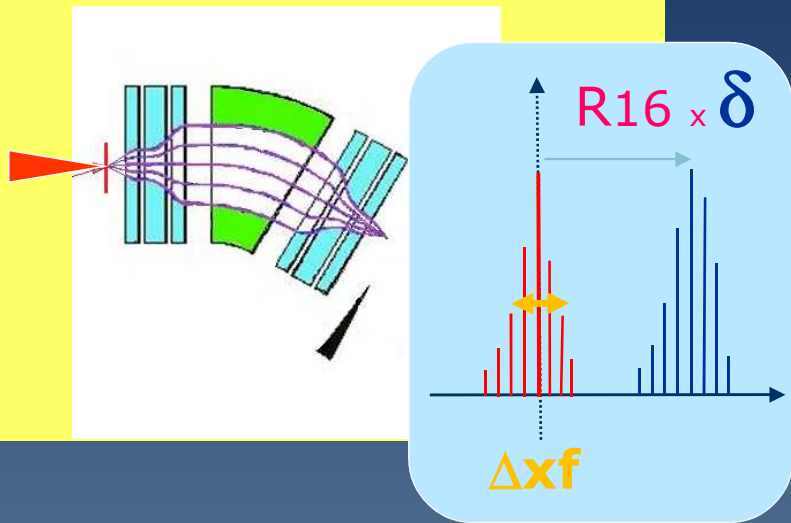
Maximal intensity of incident primary beam

- thermal limit on target (rotative or not,....)
- maximal intensity on detection sytem
- beam losses in spectro (electrostatic sparking,....)
- radioprotection

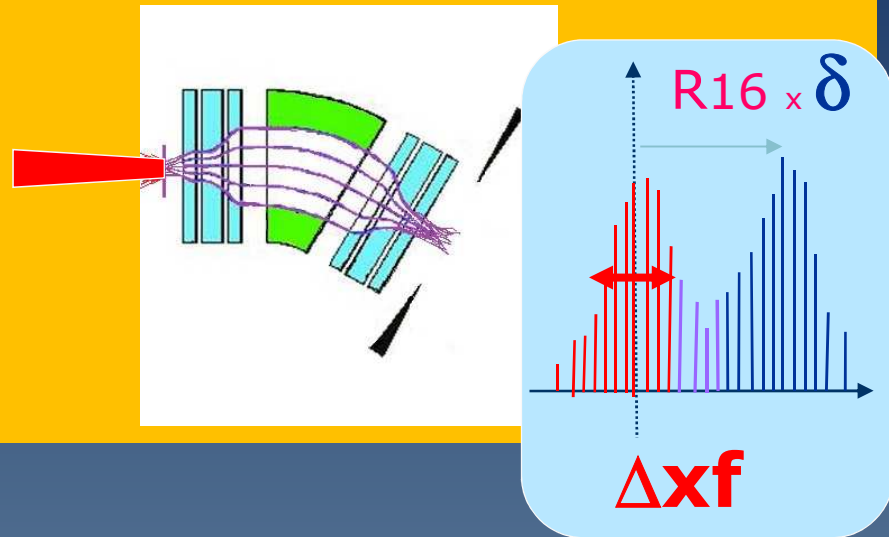
More on Fragment separators

how to optimise selection in separator

Small spot : $\Delta x_0 = \pm 1\text{mm}$



big spot : $\Delta x_0 = \pm 5\text{mm}$



The spot size Δx_{target} on target defines the beam size at focal plan

$$\Delta X_{\text{focal}} = R_{11} \cdot \Delta x_{\text{target}}$$

Big spot on Target Decrease the selection (Worse resolution)

$$\text{Resolution} = 4 \Delta X_{\text{focal}} / R_{16} = 4 R_{11} \cdot \Delta x_{\text{target}} / R_{16}$$

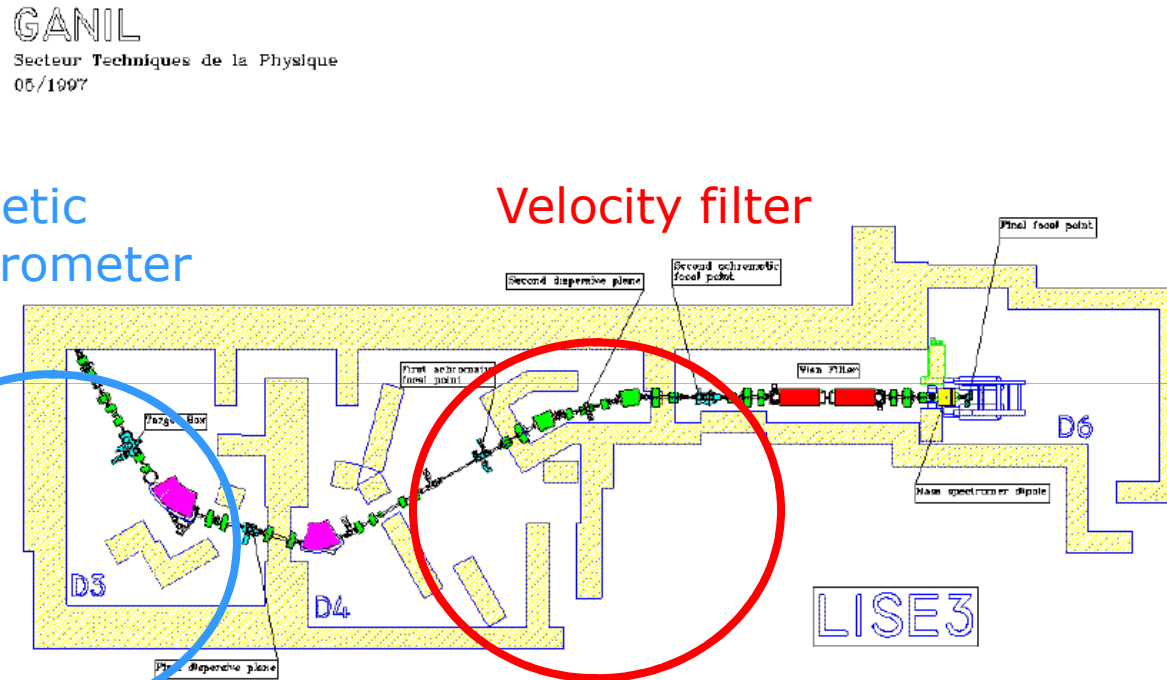
CHECK FOCUS ON TARGET (Δx_{target} small !!)

More on Fragment separators

LISE separator with Wien filter (ganil)

Magnetic spectrometer

Velocity filter



Specifications

$L=35\text{m}$

$B\rho_{1\text{max}} = 4.3\text{Tm}$ // $B\rho_{2\text{max}} = 3.2\text{Tm}$

$\Delta p/p = \pm 2.5\%$

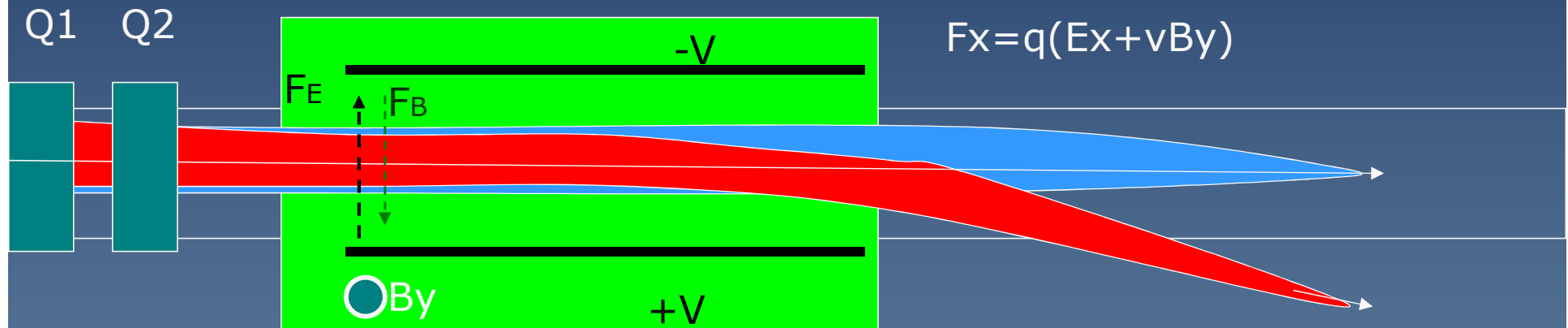
$\Delta x' = \pm 20\text{mrad}$

$\Delta y' = \pm 20\text{mrad}$

The velocity filter (so-called **Wien filter**)

The wien filter use Electric field
+ magnetic field

$$F = F_E + F_B = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$



The particles with $v_0 = -E_x/B_y$ are not deflected ($F=0$)

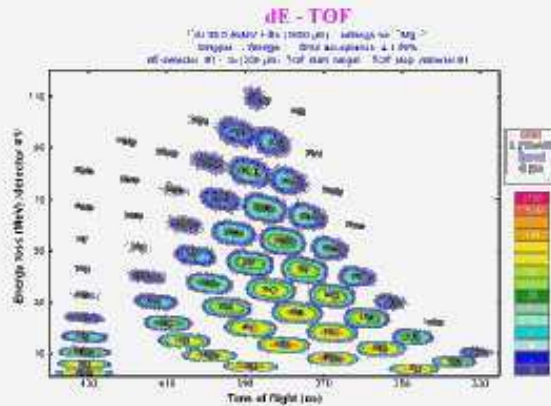
The particles with a Large velocity deviation ($v \neq v_0$) are deflected

Nota : trajectories in Electric field depend on the « electric rigidity » of the particles : $E\rho = \gamma MV^2/Q$

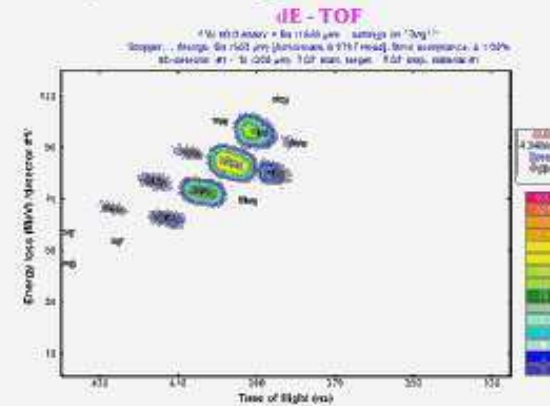
$$\frac{1}{\rho_{trajectory}} = \frac{E_x}{E\rho} - \frac{B_y}{B\rho}$$

LISE separator with wien filter

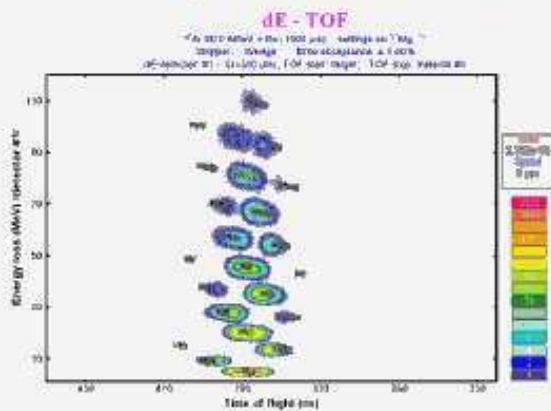
$B\rho$ selection



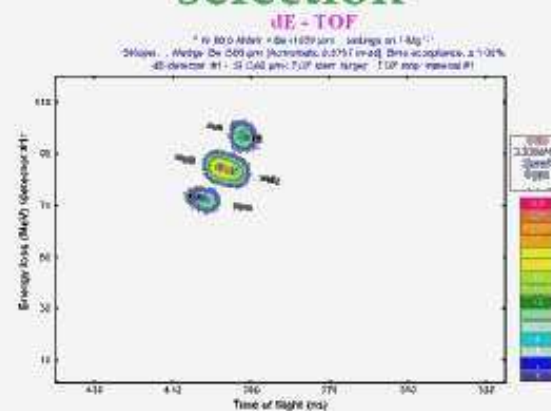
$B\rho$ +wedge selection



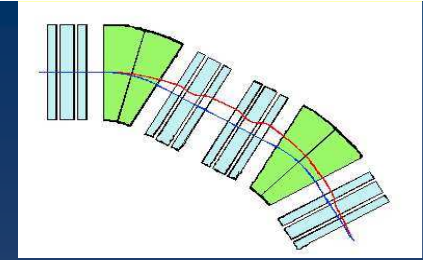
$B\rho$ +velocity filter selection



$B\rho$ +wedge+velocity filter selection



How to get a Fragment separator achromatic



Trajectories are Independent from δ (achromatic)

$$\text{IF } R_{16}(A+B)=0$$

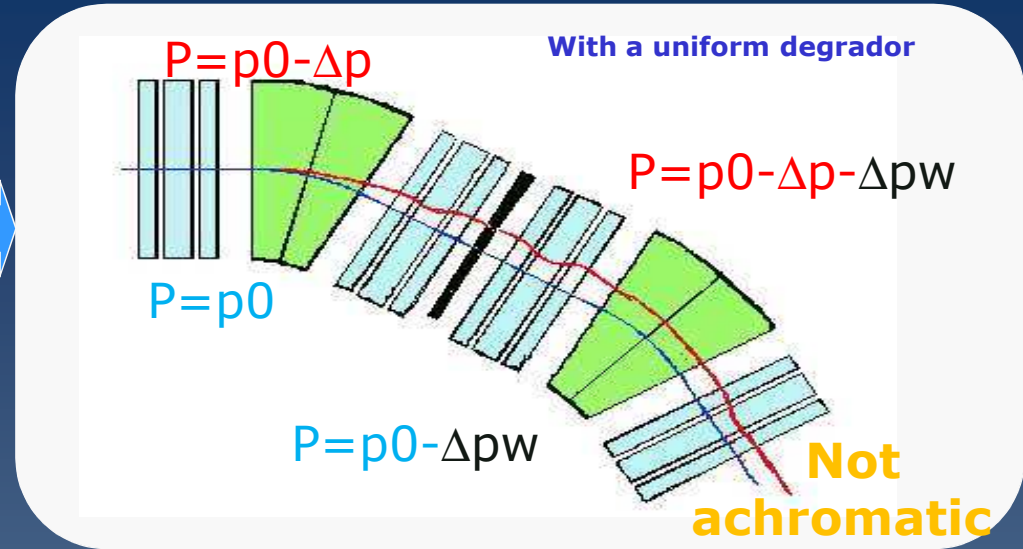
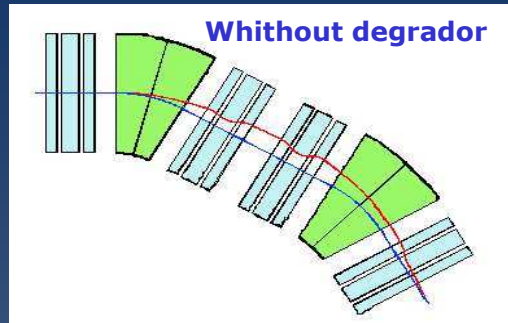
Dipole geometry **and** quad setting are adjusted to get $R_{16}(A+B)=0$

$$R(A+B) = R(B) \times R(A) = \begin{bmatrix} R_{11}^B & 0 & 0 & 0 & 0 & R_{16}^B \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ & & & & L^B & \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} R_{11}^A & 0 & 0 & 0 & 0 & R_{16}^A \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ & & & & L^A & \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Achromaticity if $R_{16}(A+B) = R_{16}(B) + R_{16}(A) R_{11}(B) = 0$

1) Why a degrador (wedge) is not uniform in x :

2 particles
With same
 Z , but
different
 $B_p = P/q$



Goal of the degrador :

All the same particles (Z, A) should re-focus at end of the B stage whatever their B_p (δ) => achromatic degrador (Wedge) : $R_{16}(A+B)=0$

Adding a uniform degrador makes the optics chromatics at the End

Before degrador δ_A , the momentum deviation of the 2 trajectories is $\delta_1 = -\Delta p/p_0$

After degrador $\delta_B = [p_0 - \Delta p - \Delta p_w - (p_0 - \Delta p_w)] / [p_0 - \Delta p_w]$

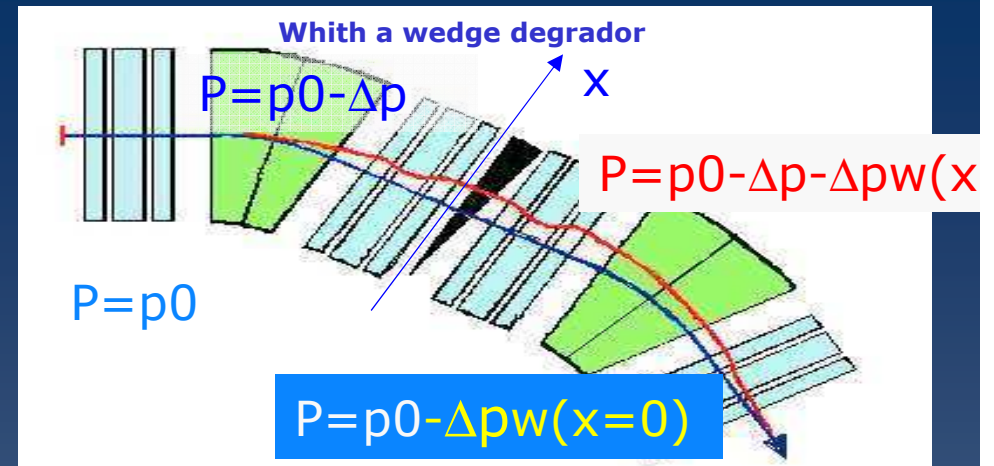
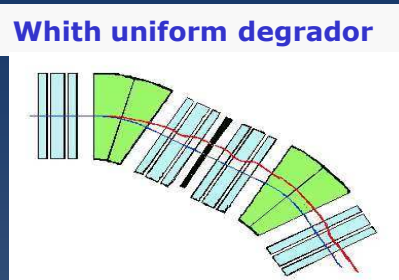
$$\delta_B = [-\Delta p] / [p_0 - \Delta p_w] \quad \delta_A \neq \delta_B$$

if the Optics is achromatic without degrador ($\delta_A = \delta_B$)

Optics will not be achromatic with a uniform degrador with $\delta_A \neq \delta_B$

2) Why a degrador (wedge) in not uniform in x

2 particles
With same Z,
but different
 $B_p = P/q$



Optics will be achromatic with degrador if ($\delta_A = \delta_B$)

The solution for having $\delta_A = \delta_B$: degrador thickness $T = T(x)$

Proof :

Before degrador, the B_p deviation of the 2 trajectories

$\delta_A = -\Delta p/p_0 = x_A/R_{16}$ after the wedge (degrador) $P \Rightarrow P - \Delta p_w$

After degrador

$$\delta_B = [p_0 - \Delta p - \Delta p_w(x_A) - (p_0 - \Delta p_w(x_A=0))] / [p_0 - \Delta p_w(x_A=0)]$$

$$= [-\Delta p - \Delta p_w(x_A) + \Delta p_w(x_A=0)] / [p_0 - \Delta p_w(x_A=0)]$$

$$(\delta_A = \delta_B) \Rightarrow \Delta p_w(x_A) \# \Delta p_w(x=0) \times [1 + x_A / R_{16}(A)]$$

$$\text{Thickness of the Wedge} \# K_x [1 + x_A / R_{16}(A)]$$

Beam emittance : (# optical quality)

The **emittance** is a **volume of phase space** occupied by a beam

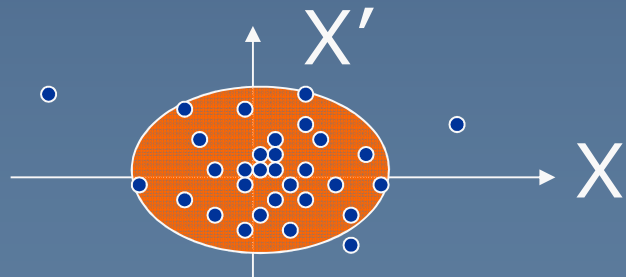
6 Dimensions

For practical reasons we use the subspace measurement (x,x') & (y,y')

Horizontal Emittance : area in (x,x')

Vertical Emittance : area in (y,y')

Longitudinal Emittance : area in (energy ,time)



$$\varepsilon_{rms} = 4(\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2)^{1/2}$$

ε = area of the ellipse ,which

correspond to x% particles

81

Liouville theoreme : emittance is conserved in a beam line..

Example n°1: fragments separator @Riken(Japan)

E#300-500 MeV/A L=77m

6 dipoles magnets, 42 quadrupole magnet

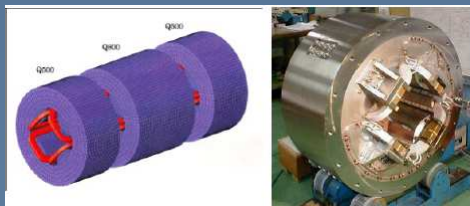
Suppression of the primary beam

(many dipoles, degrator selection)

Help the selection of very rare nuclei

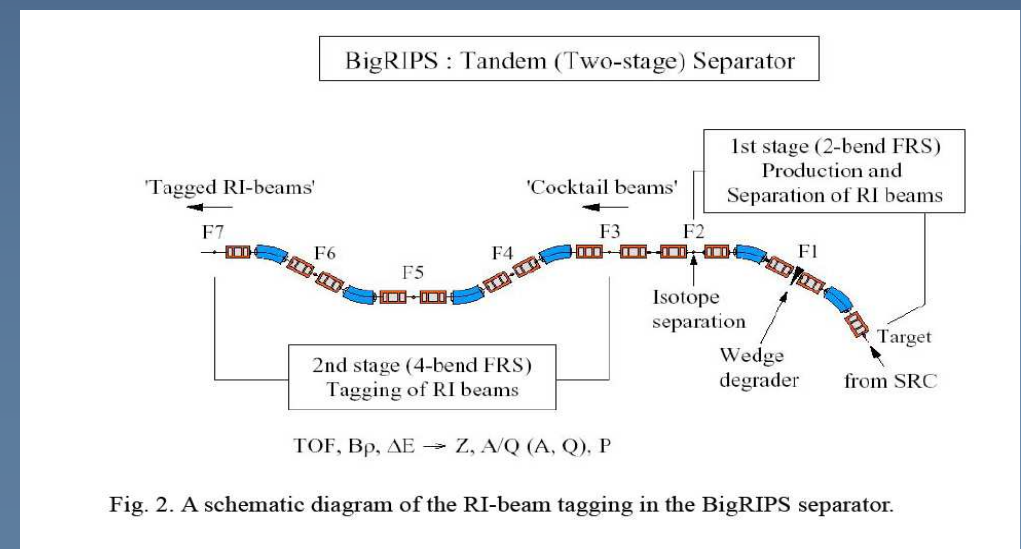
Selection of 4-5 nuclei

Identification (DE-TOF)



Superferric quads

B.Jacquot// Ganil

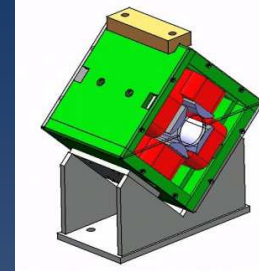
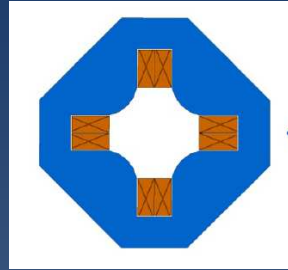


Quadrupole technology

1 : Normal conducting quad

hyperbolic pole (Fe)
coils (Cu)

$G \sim 10$ Tesla/m



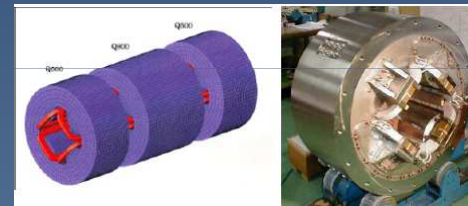
Larger Aperture
or/
and
Higher strength

2 : Superferric quad

hyperbolic pole (Fe)
coils (NbTi)

Higher Gradient , larger aperture
possible (A1900, BigRips, Synchro.)

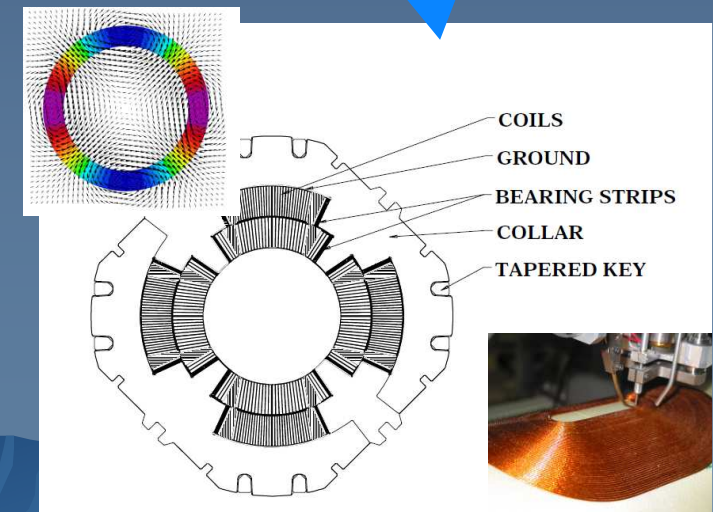
$G \sim 20-30$ Tesla/m



3 : Superconducting quad

No pole !!!!!
 $\cos(2\theta)$ coils (NbTi)

$G \sim 40-200$ Tesla/m (Cern LHC...)



Example n°2: VAMOS Spectrometer

L=8 meters, 1 dipole, rotative platform

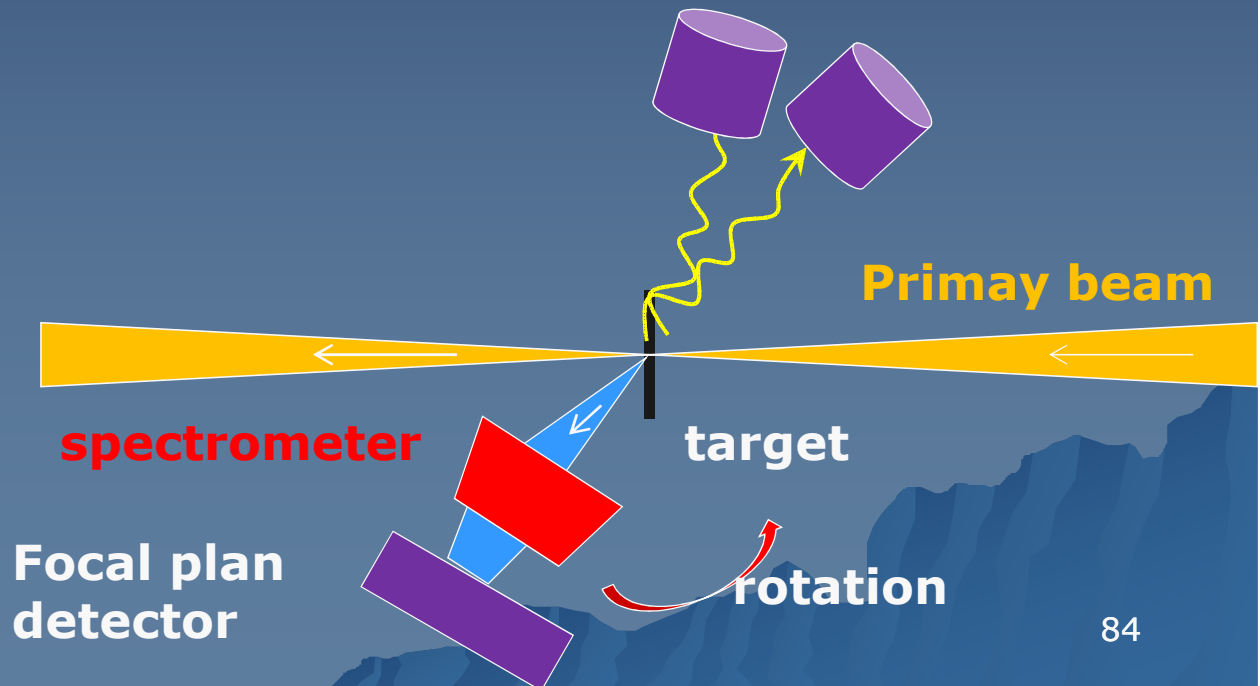
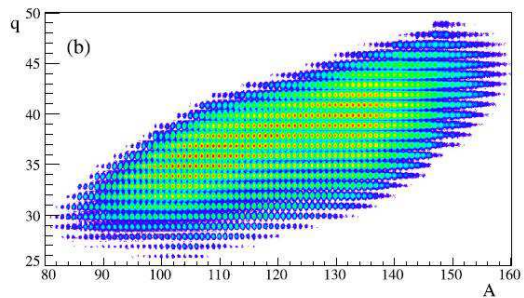
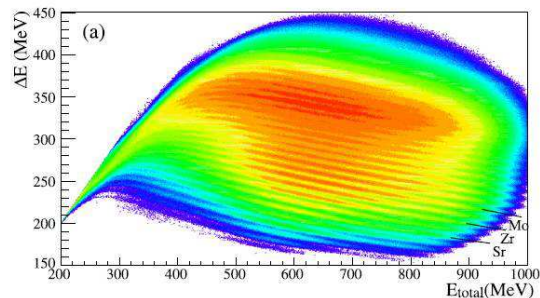


Suppression of the primary beam
(by rotation)

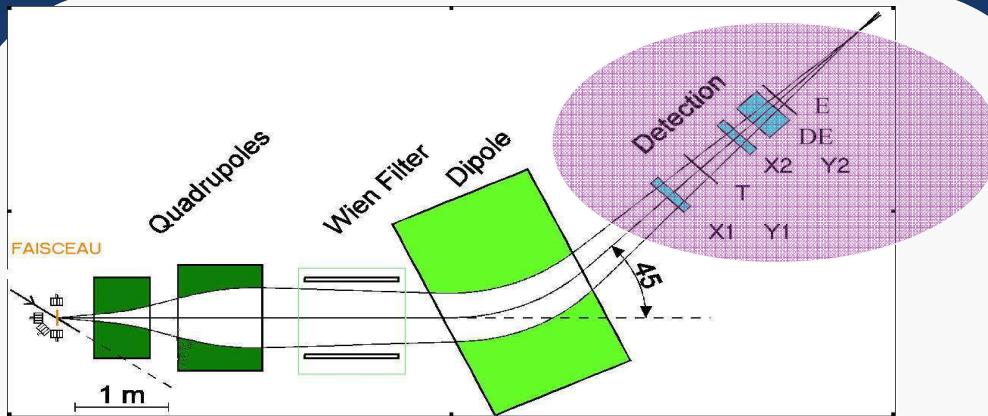
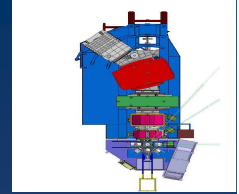
Selection of 20-300 nuclei

Help Identification (ΔE -TOF,
position and angle measurements)

300 fission fragments id.



Example n°2: VAMOS Identification



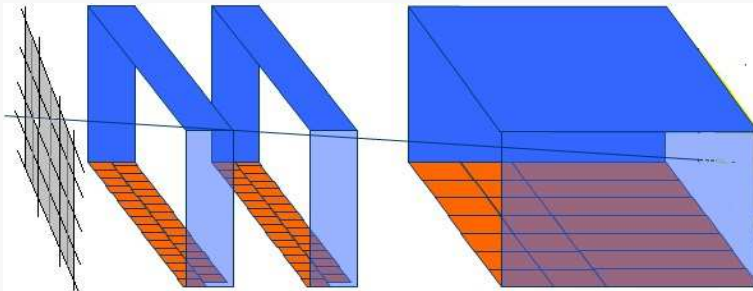
In the focal plane, 7 quantities are measured :
T, **x1**, **y1**, **x2**, **y2**, **ΔE**, **E**

T : Multi Wire PPAC

x1, y1
x2, y2 :

$$x' = (x_1 - x_2) / d = \tan(\theta)$$

$$y' = (y_1 - y_2) / d = \tan(\phi)$$



MWPPAC

(**Tof**)

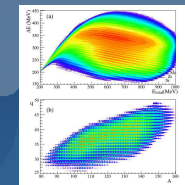
Drift Ch.
(X, X', Y, Y')

Ionis. Ch.
(ΔE, E)

ΔE, E : **ionisation CHAMBER**

$$B\rho = B \rho_0 (1 + x / R_{16} + a x'^2 + b x^2 + c x^3 + \dots)$$

Equation is non-linear in x, x', y, y' (Aberrations)



Non linear effects in optical system

1rst order

$$\vec{Z}_2 = R \cdot \vec{Z}_1 + \dots \varepsilon$$

for large angle, large B_p deviation 2nd order, third order is required.

$$Z_{2i} = \sum_{j=1}^6 R_{ij} \cdot Z_{1j} + \sum_{k=1}^6 \sum_{j=1}^6 T_{ijk} Z_{1j} \cdot Z_{1k} + \dots$$

1rst order

2nd order

Linear Approximation holds for small angle, small B_p deviation... (#30mrad, $\delta < 2\%$)

$$Z_1 = (x, x', y, y', l, \delta)_1$$

Effects of second order :

- Inclination of focal plane
- the Focusing strenght of quads is b_p dependant
- Large angle particles are not well focused

Non linearities (ABERRATIONS) come

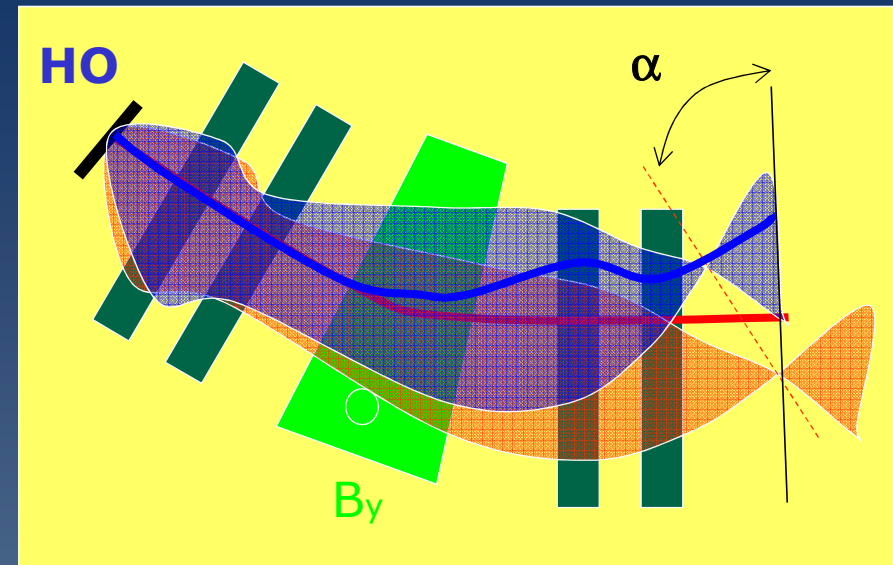
- with large acceptance (large x' and large δ)
- but also, with field defects in quads and dipoles

Non linear effects in optical system

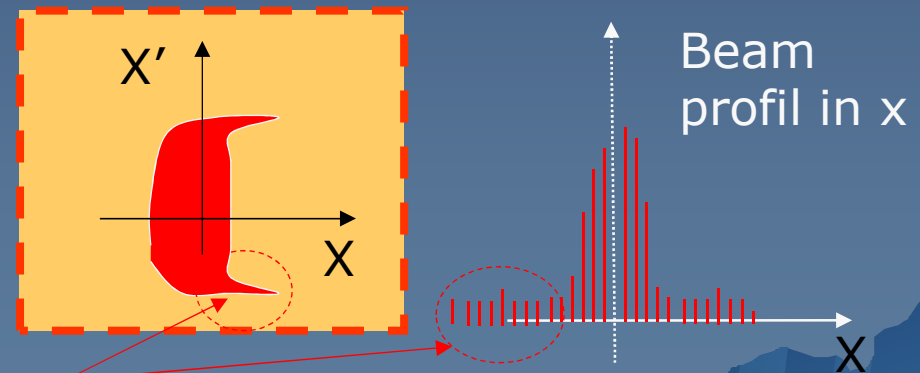
Ex1: Inclination α of the focal
in a spectrometer

$$\text{tg}(\alpha) = R_{16} / T_{126} \cdot R_{11}$$

- Choice of the **dipole Angle**
- Magnetic sextupole has to be used for correction



Ex2: distorsion of beam ellipse
In phase space
Inducing Distribution wings



Optical aberrations (non linearities)

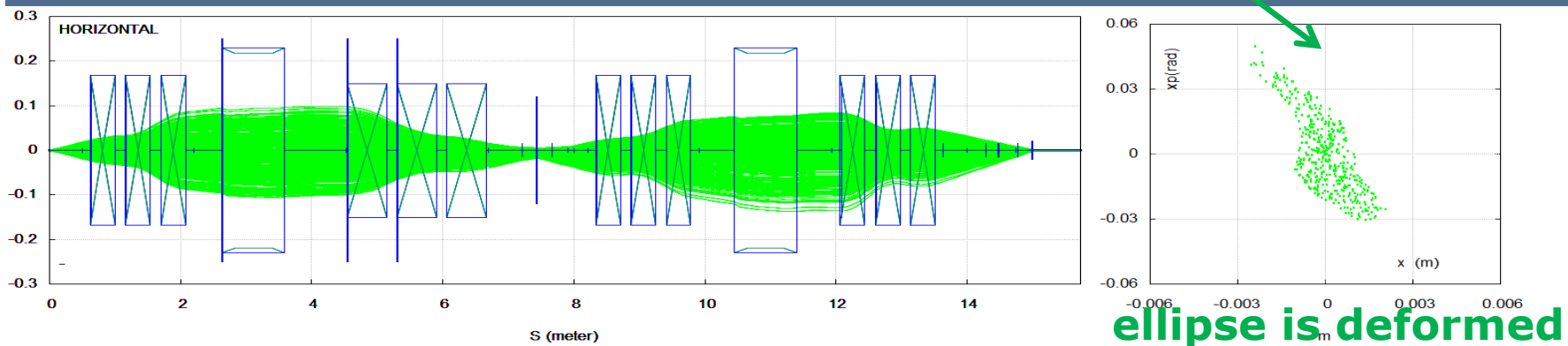
Non linear effects in optical system

Beam optics is linear when $x < 5\text{cm}$
 $x' < 30\text{mrad}$
 $\delta < 2\%$

Beam is a nice ellipse in phase space, R matrix is sufficient

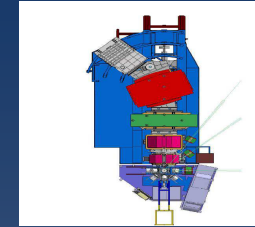
If $|X'| > 30\text{mrad}$ or $|\delta| > 2\%$
Beam are not well represented by an ellipse

R matrix is **not sufficient** for the calculation
(field maps + tracking with « Runge kutta » simulation needed)

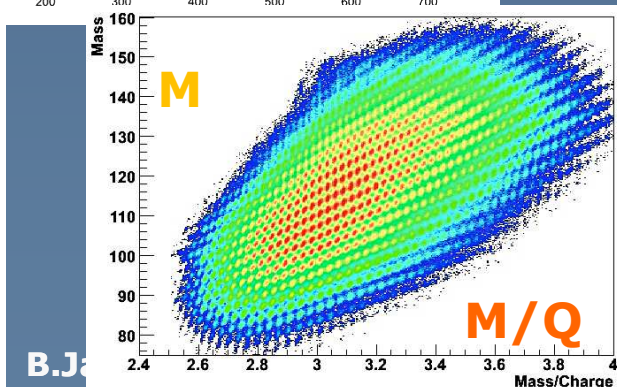
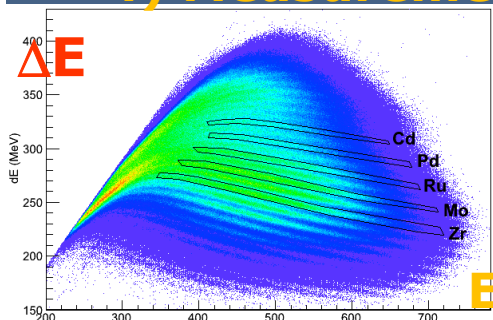


Example n°2: VAMOS Spectrometer

Particle identification Method (M, q, Z)



- 1) Measurement of the time of flight (TOF) => **velocity**
- 2) Measurement of the position x_{focal} after the spectrometer
=> $B\rho = B \times R_{dipole} (1 + x/R_{16} + \dots)$
- 3) Measurement of the energy loss ΔE in a thin detector (Ionization Chamber)
- 4) Measurement of residual energy E_r ($E_{kinetic} = (\gamma - 1)M c^2$)



v
 M/q
 Z
 M_1

$$v = T_{flight} / L_0$$

$$M/q = B\rho / \gamma v$$

$$Z \# k \Delta E \dots$$

$$M_1 = (E_r + \Delta E) / [c^2 (\gamma - 1)]$$

finally

$$Q = M_1 / [M/q]$$

$$M = [M/q] \cdot Q$$

$$Z \# k(E) \Delta E$$

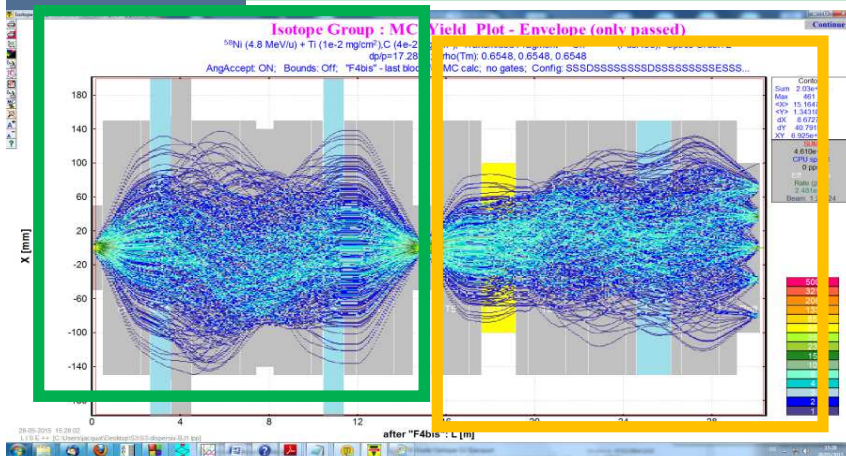
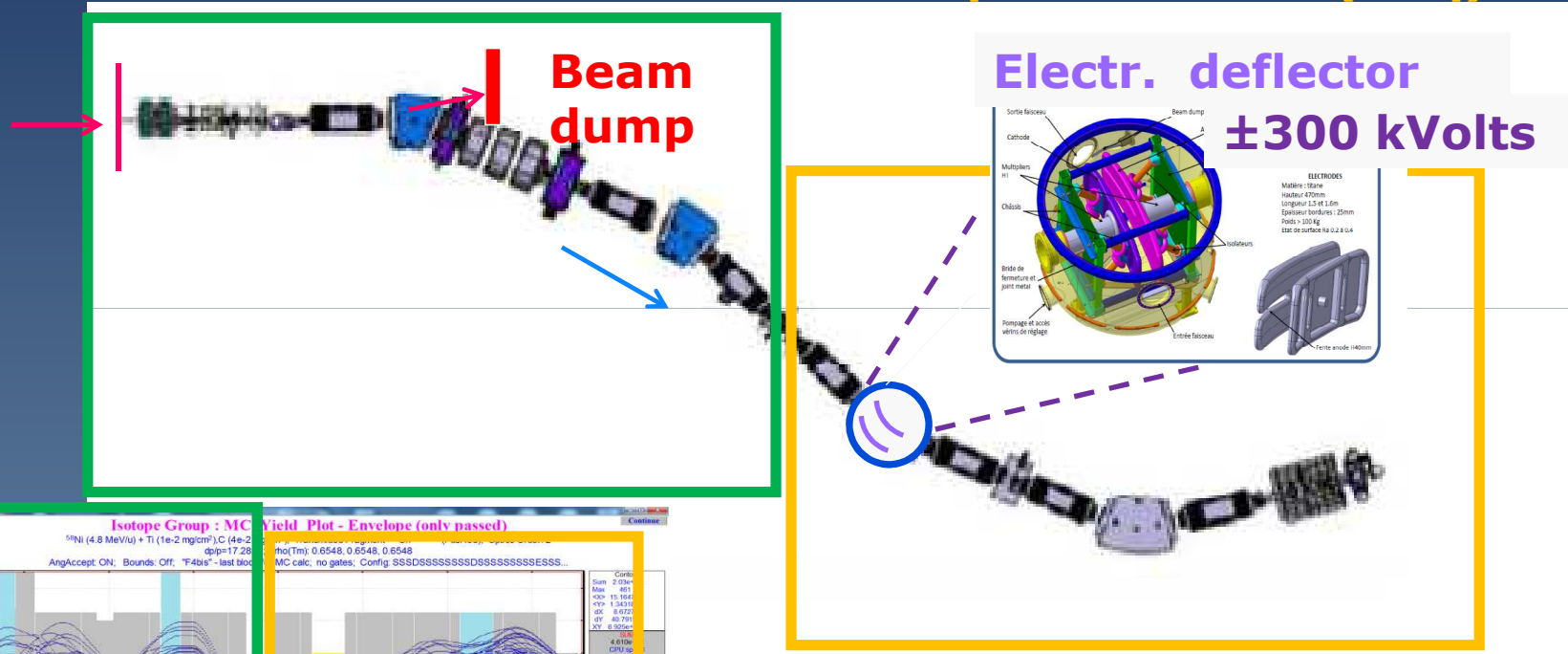
Example : S3 spectrometer @Ganil

S3 :

1 Magnetic achromatic separator (2 dipoles)

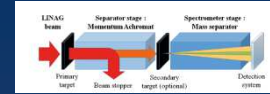
+

1 mass spectrometer (M/q)



Lise++ simulation :
fusion reaction , 5 charge states
(horizontal plane)

Example : S3 spectrometer @Ganil

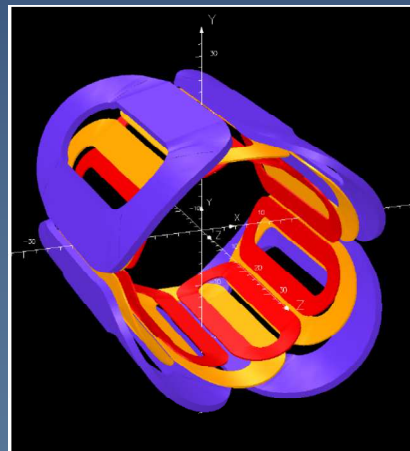
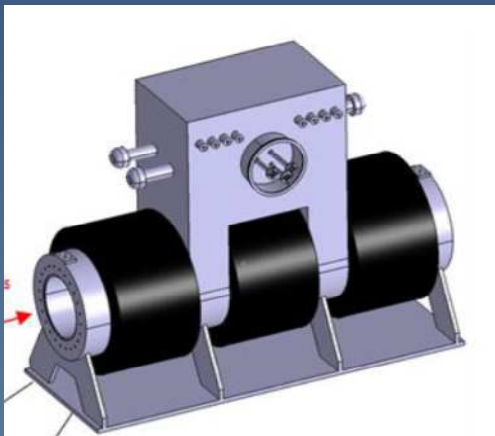
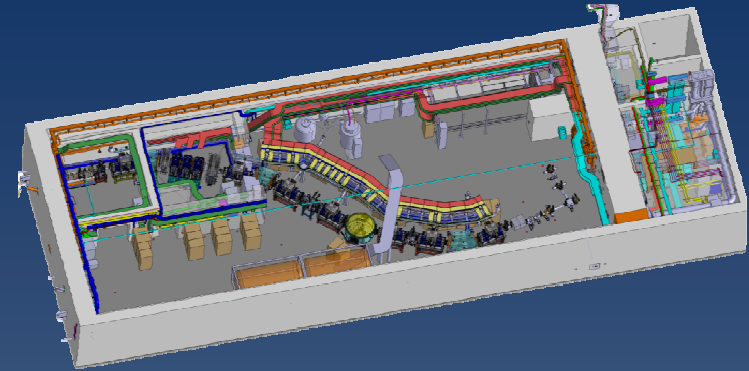


S3 :

1 Magnetic achromatic separator (2 dipoles)

+

1 mass spectrometer (M/q)



Superconducting quadrupole triplet : Coil (NbTi), without pole

- Superconducting coils with multipolar corrections (hexapole+octupole)

- Quadrupoles : Aperture very large =0.15m ;